Appendix B

(not for publication, available at http://www.severinov.com/AppendixB_mechanisms.pdf)

Part 1. Relaxing the assumption $p_{i,j}(\theta_i, \theta_j) > 0$ for any $\theta_i \in \Theta_i, \theta_j \in \Theta_j$, i and j.

If this assumption does not hold, we proceed as follows. First, introduce the following definition.

Definition 5 *$T \equiv \{T^1, ..., T^q\}$ is the finest partition of the set $\{\theta \in \Theta | p(\theta) > 0\}$, i.e. the set of type profiles occurring with a positive probability, which satisfies the following property **FP**: For all i and $\theta_i \in \Theta_i$, there exists $l \in \{1, ..., q\}$ s.t. $\{(\theta_i, \theta_{-i}) | \theta_{-i} \in \Theta_{-i}, p(\theta_i, \theta_{-i}) > 0\} \subset T^l$.*

T is well-defined because the trivial partition $\tilde{T}$ s.t. $q = 1$ and $\tilde{T}^1 = \{\theta \in \Theta | p(\theta) > 0\}$ satisfies property **FP**, and a meet of two partitions T' and T'' possessing property **FP** is also a partition satisfying this property. In fact, if for any pair of types θ_i and θ_j of any two agents i and j, there exists $\theta_{-i-j} \in \Theta_{-i-j}$ s.t. $p(\theta_{-i-j}, \theta_i, \theta_j) > 0$, then T is trivial, i.e. $q = 1$.

Also, note that since $p_i(\theta_i) > 0$ for all i and $\theta_i \in \Theta_i$, the projection of partition T on type space Θ_i is a partition of Θ_i which we denote by $\{T_i^1, ..., T_i^q\}$.

Definition 6 *The decision rule $x(\theta)$ is ex-ante socially rational for prior $p(.)$ ($EASR(p)$) if*

$$\sum_{\theta \in T_l} \sum_{i=1}^{n} u_i(x(\theta), \theta) p(\theta) \geq 0 \quad \text{for all } l \in \{1, ..., q\} \tag{54}$$

Then it is easy to establish the following result:

Lemma 4 *If the allocation profile $(x(\theta), t(\theta))$ is incentive compatible and interim individually rational, then $x(\theta)$ is $EASR(p)$, i.e. ex ante socially rational for prior $p(.)$.*

Proof Consider any agent i and θ_i s.t. $\theta_i \in T_i^l$ for some $l \in \{1, ..., q\}$, i.e. $(\theta_i, \theta_{-i}) \in T^l$ if $p(\theta_i, \theta_{-i}) > 0$. By Property **FP**, $IR(\theta_i)$ is equivalent to the following:

$$\sum_{\theta_{-i}:(\theta_i, \theta_{-i}) \in T^l} \left(u_i(x(\theta_{-i}, \theta_i), (\theta_{-i}, \theta_i)) + t_i(\theta_{-i}, \theta_i) \right) p(\theta_{-i}, \theta_i) \geq 0 \tag{55}$$

Summing (55) over all $\theta_i \in T_i^l$ and then adding the resulting inequalities together for all $i \in \{1, ..., n\}$, we obtain $\sum_{\theta \in T^l} \sum_{i \in \{1, ..., n\}} \left(u_i(x(\theta), \theta) + t_i(\theta) \right) p(\theta) \geq 0$. Budget-balancing then implies (54). ■

Careful reading of the proofs confirms that in this case Theorem 1 holds for any $EASR(p)$ decision rule. This property is required to establish modified Lemma A3 in the proof. Furthermore, Theorem 1 holds with the following modification: the expected social surplus conditional on a particular element of the partition T can be allocated in an arbitrary way to the agent-types within this element of the partition.