# Strategic Information Acquisition and Transmission

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*This paper explores the implications of costly information acquisition in a strategic communication model. We show that equilibrium decisions based on a biased expert's advice may be more precise than when information is directly acquired by the decision maker, even if the expert is not more efficient than the decision maker at acquiring information. This result bears important implications for organization design. Communication by an expert to a decision maker may often outperform delegation of the decision making authority to the expert, as well as centralization by the decision maker of both information acquisition and decision making authority.*
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Strategic information transmission is one of the central topics in economics of information. Starting from the seminal work of Crawford and Sobel (1982), this literature highlights the limited scope of information transmission via cheap talk messages, which generically leads to inaccurate or imprecise decisions (see Austen-Smith (1993), Gilligan and Krehbiel (1987, 1989), Krishna and Morgan (2001), Wolinsky (2002), Battaglini (2002), Ambrus and Takahashi (2008)). A common assumption in this literature is that perfect information is exogenously given to the sender for free. The exceptions include Austen-Smith (1994), Ottaviani (2000) and Ivanov (2010). In Austen-Smith (1994), the sender may either acquire complete information or remain ignorant. In Ottaviani (2000), the amount of information available to the expert is exogenous. In Ivanov (2010), informational structure can be selected costlessly by the decision-maker.

However, in reality information is typically obtained through time-consuming and costly research effort.[1] This being our point of departure, we study a model of strategic communication in which information is costly and the decision to acquire it is taken endogenously. In this setting, we demonstrate that the decision-maker can induce the expert to acquire more information than the decision-maker would acquire directly, even when the expert and the decision-maker have the same technology of information ac-

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[1] For example, this is the case for investment advice by financial analysts, for policy advice by experts reporting to Congress, for expert witnesses in trials, as well as for many other real-world applications of strategic information transmission games.

quisition. This result provides a foundation for our main finding: the decision-maker can take more precise actions when the latter are based on the advice of a biased expert -provided that the bias is sufficiently small- rather than on the decision-maker's direct information acquisition. This stands in contrast to the "common wisdom" of the extant literature that the decisions based on the advice of a biased expert suffer from a loss of precision.

To explain our results, let us first highlight the main features of our model. Initially, both players – the decision-maker and the expert – are uninformed about the state of the world and share a common prior. Information about the state of the world can be acquired by performing "experiments" or "trials." The cost and precision of the acquired information is measured by the number of the performed "trials."[2] This model of information acquisition is simple and tractable, and fits well as a description of a number of real world situations such as aggregation of individual opinions from sincere voting, surveys, or experiments. Moreover, as we explain below, the main driving forces identified in our analysis are quite general and extend to other settings and different -discrete or continuous- models of information acquisition.

In our baseline model, the expert acquires the information and then conveys a cheap-talk message to the decision-maker, who then takes an action. We consider two scenarios: overt information acquisition and covert information acquisition.[3] In the former, the decision maker observes the quantity of information acquired by the expert, but not its content. In the latter, the decision-maker observes neither the quantity nor the content of the expert's information. In both cases, we focus on the amount of information acquired and credibly transmitted by the expert, which translates into the precision of the final action taken by the decision-maker. We then compare the outcomes of these two communication games against two alternatives: the first one is direct information acquisition by the decision-maker, the second one is delegation to the expert of both information acquisition and the choice of action.

The expert's overinvestment in information acquisition is driven by different forces in the overt and covert games. In the overt game, the expert overinvests in order to avoid the negative implications of the decision-maker reacting to the expert's deviation at the information acquisition stage. The worst credible punishment that the decision-maker can inflict on the expert in case of such a deviation -which of course provides the strongest incentives for the information acquisition- is to ignore the expert's message, unless the expert acquired the "right" amount of information. In technical terms, a babbling equilibrium is played off the equilibrium path. [4]

Moreover, the focus on off-path play of babbling equilibria is well-grounded and motivated in reality. Specifically, in a number of situations the decision-makers only heed advice of experts whose qualifications or effort exceed the threshold set by the former. Consider for example expert witnesses in legal trials. In the U.S., the Federal Rules of

[2] Our set-up is related to the Bernoulli-Uniform model of cheap talk analyzed by Morgan and Stocken (2008).

[3] We refer to these two scenarios as "overt game" and "covert game", in the remainder of the paper.

[4] Ubiquitous in communication games, a babbling equilibria involves the decision-maker taking a decision unaffected by the expert's message, and, thus, the expert is indifferent among sending any message, and adopts a completely uninformative communication strategy.

Evidence specify that testimony by an expert witness is acceptable only if it "...is the product of sufficient facts or data," and "is the product of reliable principles and methods [5]". This rule is sufficiently broad and allows the judge to tailor her threshold of acceptability to the particular case under consideration.[6] If the judge finds that an expert has not met this threshold, (s)he would typically disqualify the expert rather than allow a limited testimony by the expert. Our results suggest that this legal procedure provides a powerful incentive for information acquisition.

Other examples of what essentially is a threshold knowledge rule for admissibility of an expert's advice can be found in politics (parliamentary and congressional hearing making use of expert's advice), financial and consumer markets (financial advisors and real estate agents have rating systems and certain customers will only deal with the agents and advisors who have the highest rating category[7]), and academia (short reference letters that do not describe in detail an academic's research are usually disregarded by hiring and tenure committees).

Furthermore, we also identify a larger parameter region in which our strict overinvestment result holds in all but one Pareto efficient equilibria of the overt game. The only exception is the expert's ex-ante preferred equilibrium in which our result holds weakly: the expert acquires and reveals exactly as much information as the decision-maker would acquire directly. This equilibrium outcome can be sustained playing the most informative communication equilibrium both on and off path. In any other Pareto efficient equilibrium, the final decision is strictly more precise than the decision that would be made by the decision maker acquiring information directly. This result does not rely on the threat of babbling off-path.

In the covert game, the information acquisition investment is unobservable, and hence the decision-maker cannot punish the expert by tailoring her behavior to the actual amount of information acquired. In fact, we establish that when searching for the most informative and/or Pareto efficient equilibria and characterizing attainable levels of information acquisition, there is no loss of generality in focusing on equilibria in which the expert does not communicate how much information he has acquired, as the latter would be a non-verifiable "cheap-talk" message. So, the decision-maker interprets any expert's message under the belief that the latter has acquired the equilibrium amount of

[5] According to the Federal Rule of Evidence 702:

"A witness who is qualified as an expert by knowledge, skill, experience, training, or education may testify in the form of an opinion or otherwise if: (a) the expert's scientific, technical, or other specialized knowledge will help the trier of fact to understand the evidence or to determine a fact in issue; (b) the testimony is based on sufficient facts or data; (c) the testimony is the product of reliable principles and methods; and (d) the expert has reliably applied the principles and methods to the facts of the case."

[6] Berlin and Williams (2000) report that a case in which: "...The Illinois Supreme Court then pointed out that it is the judge who must determine whether a potential expert witness is qualified to render opinions in a specific lawsuit" They quote the opinion of said Court in the case *Jones v. O'Young et al.* as follows: "...The trial court has the discretion to determine whether a physician is qualified and competent to state his opinion as an expert regarding the standard of care. . . .By hearing evidence on the expert's qualifications and comparing the medical problem and the type of treatment in the case to the experience and background of the expert, the trial court can examine whether the witness has demonstrated a sufficient familiarity with the standard of care practiced in the case. . . [If the expert witness does not satisfy these requirements], the trial court must disallow the expert's testimony. . . . The requirements are a threshold beneath which the plaintiff cannot fall without failing to sustain the allegations of his complaint."

[7] J.D. Power and Associates system of rating for brokers provides one example.

information even if that is not the case. We refer to this property as inflexibility of the equilibrium language. Importantly, this inflexibility is *an equilibrium property* in our model, not an assumption.

The inflexibility of the equilibrium language reduces the profitability of the expert's deviations in information acquisition. Specifically, the equilibrium language determines the set of final decisions which the expert can induce the decision-maker to take. This set of actions is particularly well tailored to the equilibrium amount of information. On the other hand, by the inflexibility property, this set of feasible actions does not change with the amount of information actually acquired by the expert, and hence it is less suitable to the non-equilibrium quantity of information. This results in a lower precision of the final action, hurting the expert when he deviates. This effect is less powerful than the off path punishment in the overt game. Therefore, stronger conditions on the parameters are required for the overinvestment to occur. As in the overt game, the strict overinvestment result extends to all Pareto efficient equilibria of the covert game, except the equilibrium preferred by the expert which is characterized by weak overinvestment.

Examples of fixed communication language are fairly common in economic environments. In particular, the language of financial advice is often standardized. Standard and Poor's Capital IQ equity analysts rank assets on a qualitative 5-point scale (Strong Sell, Sell, Hold, Buy, Strong Buy). Similarly, consumer research firms, such as Consumer Report, J.D. Powers and Associates and others, typically rate the quality of products on a grid with a fixed number of points. Standardized restricted communication protocols can be found in public administration and in the military. In these examples the adopted languages/grids, although endogenous, are apparently not sensitive to the amount of information possessed by the sender and may not be suitable when too much or too little information is acquired.

Finally notice that in both the overt and covert games, the expert's overinvestment in information acquisition is not beneficial to the decision-maker by itself, but only when the loss of the acquired information is transmission is not too large. In turn, the small loss in transmission is possible only when the expert's bias (the misalignment of interests between the expert and the decision-maker) is small. Hence, the expert's bias must be sufficiently small for our results to hold.

Our analysis has significant implications for the theory of optimal organization. A number of authors have cast doubt on the optimality of communication-based organizations vis-a-vis the alternatives. In particular, Dessein (2002) and Ottaviani (2000) have shown that a communication-based organization, in which the principal's decisions are based on the advice of a biased expert with access to perfect and free information, is dominated by delegation of the decision-making authority to the expert due to the loss of information in transmission. Similar results in somewhat different frameworks have been established by Aghion and Tirole (1997) and Gilligan and Krehbiel (1987).[8]

In contrast to these authors, our paper shows that, when information acquisition is

[8]Mitusch, and Strausz (2005) is an interesting contribution that studies how and when adding a mediator can facilitate communication between the decision-maker and an informed party, and thus a three level hierarchy can outperform a two-level one.

added to the set of organizational tasks, communication-based organization performs better than either delegation to the expert of both activities or direct information acquisition by the decision-maker. Thus, our results provide support for the prominent role of information transmission between experts and decision-makers in organizations, which has been postulated theoretically and confirmed empirically (see, e.g., Bolton and Dewatripont,1994, and Garicano, 2000).

Viewed from another perspective, our paper suggests that it is optimal to divide the tasks of information acquisition and decision-making in an organization when the conflict of interests within the organization is small. Our strict overinvestment result implies that such division of labor makes the searching player exert more effort, while combining both information acquisition and decision-making tasks in the hands of a single party results in less search effort, and lower efficiency because of the positive externality on the other(s). This is particularly relevant to partnerships. Empirically, it is in line with the findings in Nelson (1988), who documented significant task differentiation among the lawyers within law firms. Some deal mostly with information acquisition tasks (taking depositions, research, gathering information from clients), while others focus on operational and decision-making roles such as developing case strategy, preparing and arguing motions and negotiating with the opposing parties. While this division of labor within large firms may reflect the distinction between partners and associates, this does not play a role within smaller law firms.[9]

## I. Literature on Information Acquisition.

The study of information acquisition has largely been unexplored in the strategic communication literature, except for a few recent contributions. In particular, Eso and Szalay (2010) consider a game in which an expert has the same preferences as the decision-maker and is initially uninformed but can learn the exact realization of the state by paying a fixed cost. The decision-maker commits ex-ante to a message set (equivalently, action set) that the expert can choose from. It is shown that restricting this message set can induce the sender to acquire information for a larger range of costs. Similarly, Szalay (2005) shows that restricting the set of actions available to the agent in the delegation game can increases the latter's incentive to acquire information. In both these papers, the restriction on the set of messages (or actions) available to the expert is chosen ex-ante by the decision-maker, and the focus is on the normative question of which exogenously fixed language maximizes information acquisition. Our model is different in a number of significant aspects. First, unlike in those papers, in our game the language is endogenous: it does not arise as a result of a commitment but rather emerges as a feature of the equilibrium interaction between the players. Our focus is also different: we study the positive question of how much information acquisition would occur in the communication game, as well as in other organizational forms. Other substantive differences

[9]Other studies of partnerships (e.g., Farrell and Scotchmer, 1988, Garicano and Santos, 2004, Levin and Tadelis, 2005) compare them to other organizational forms, but do not delve into the matter of labor division within them. One exception is Garicano and Hubbard (2008) who study the optimal distribution of lawyers across legal fields.

between our model and those of Eso and Szalay (2010) and Szalay (2005) involve the expert's preferences and information acquisition technology: our expert is biased, he can acquire any intermediate amount of information about the true state of the world, and his information remains imprecise, except in the limit.

Another related contribution is Pei (2015) who considers covert costly information acquisition and transmission. In his model, the expert first acquires an information partition of the state space, and then observes the element of the acquired partition to which the true state belongs. His key assumption is that a sender can acquire any coarsening of a feasible partition at a lower cost. The implication of this powerful assumption is that all equilibria involve full revelation of the expert's private information. Indeed, there is no reason for the expert to purchase an information partition and then coarsen his information in transmission if, instead, he can directly purchase the corresponding coarser information partition at a lower cost and then transmit exactly what he has learned. Our information acquisition technology -via experiments which improve the precision of information- does not satisfy the assumption of Pei (2015).

Less closely related, Che and Kartik (2009) study acquisition and disclosure of verifiable information. In their model, the expert has the same preferences as the decision-maker but a different prior. Because of verifiability, an informed expert can only disclose his signal or conceal it. These authors focus on the choice of the expert by the decision-maker, and show that the latter would prefer an expert with a prior different from hers. The divergence in prior beliefs, while stifling communication, provides stronger incentives for the expert to put effort into information acquisition.

The rest of the paper is organized as follows. Section 2 introduces the model. Section 3 describes the main results for the overt and covert models. Section 4 derives the implications for organization design. Section 5 concludes. All proofs are relegated to the Appendix.

## II. The Model

Our model of cheap talk with endogenous acquisition of costly information by the expert-sender is a natural extension of the classic Crawford and Sobel (1982) model. There are two players, the expert and the decision maker. The decision-maker's payoff is given by

$$U^R(y,\theta) = -(y-\theta)^2, \tag{1}$$

where $\theta$ is an unknown state of the world and $y$ is the action taken by the decision-maker. For simplicity, we assume that $\theta$ is distributed uniformly over [0, 1], but the main forces driving our results are robust to different distributional assumptions.

The expert's payoff is given by

$$U^S(y,\theta,b) - c(n) = -(y-\theta-b)^2 - c(n), \tag{2}$$

where the bias $b \geq 0$ measures the preference discrepancy between the expert and the decision-maker and $c(n)$ is the cost of information acquisition when the expert performs $n$ trials as described below.

The game unfolds as follows. Initially, both the expert and the decision-maker have the same common knowledge prior beliefs that $\theta$ is distributed uniformly over $[0, 1]$. The expert then proceeds to acquire information by deciding on a number of binary trials to perform.[10] Each trial results either in a success or a failure, with probability of success equal to the true $\theta$. Conditional on $\theta$, the realization of each trial is independent of other trials. If the expert performs $n$ trials, he incurs the cost $c(n) = cn$ and simultaneously learns the realizations of all trials. Then he sends a message $m \in M$ to the decision maker, where $M$ is some message set. After receiving the message, the decision-maker chooses an action $y \in [0, 1]$.

For given $n$ and $\theta$, the number of successes $k$ is distributed according to the binomial distribution:

$$f(k|n,\theta) = \frac{n!}{k!\,(n-k)!}\theta^k (1-\theta)^{n-k}, \text{ for } 0 \le k \le n.$$

When $\theta$ is uniformly distributed, the distribution of $k$ is also uniform:

$$\Pr(k|n) = \int_0^1 \frac{n!}{k!\,(n-k)!}\theta^k (1-\theta)^{n-k} \, d\theta = \frac{1}{n+1}.$$

Finally, the posterior distribution of $\theta$ given $k$ successes in $n$ trials is a Beta distribution with parameters $k+1$ and $n-k+1$. Its density is given by:

$$f(\theta|k,n) = \frac{(n+1)!}{k!\,(n-k)!}\theta^k (1-\theta)^{n-k}, \text{ if } 0 \le \theta \le 1.$$

The corresponding posterior expectation of $\theta$ is $E[\theta|k,n] = \dfrac{k+1}{n+2}$.

We will distinguish between two cases in the analysis. In the *overt game*, prior to choosing an action $y$ the decision-maker observes the number of trials $n$ performed by the expert. In the *covert game*, $n$ is private unverifiable information of the expert.

### *A. The Overt Game*

A pure strategy Perfect Bayesian Equilibrium of the overt game is described by a tuple $\left(n, \{P_{n'}\}_{n' \in \mathbb{N}\cup\{0\}}, \{\mathbf{y}(P_{n'})\}_{n' \in \mathbb{N}\cup\{0\}}\right)$, where $n$ is the expert's number of trials; $P_{n'} \equiv \left(p_1^{n'}, ..., p_{\#P_{n'}}^{n'}\right)$ is the partition of the set of the expert's types $\left\{0, 1, ..., n'\right\}$ describing the information communicated by the expert after $n'$ trials; and $\{\mathbf{y}(P_{n'})\} \equiv \left(y_{p_1}^{n'}, ..., y_{p_{\#P_{n'}}}^{n'}\right)$ is the decision maker's action profile corresponding to partition $P_{n'}$.

[10] We envision a "batch" model in which the expert decides once and for all on the size of the batch (number of trials) to acquire. Changing the size of the batch along the road is too costly e.g., requires a high fixed cost.

According to this definition, if the expert performs $n'$ trials with $k$ successes, then he sends a message to the decision maker that the element $p_i$ of the communication partition $P_{n'}$ has occurred, where $k \in p_i$.[11] A *babbling partition* contains a single element. In a *fully separating partition*, each type is an element of the partition. Correspondingly, $y^{n'}_{p_i} \in [0, 1]$ is the action which the decision-maker takes after receiving a message corresponding to the element $p_i$ of the partition $P_{n'}$.

The following conditions must hold in an equilibrium:
(i) Action profile $\mathbf{y}(P_{n'})$ is sequentially rational for all $n'$ i.e. $y^{n'}_{p_i}$ maximizes the decision-maker's expected payoff given that the expert's type $k$ is in $p_i$:

$$(3) \qquad y^{n'}_{p_i} \in \arg\max_{y} \int_0^1 U^R(y,\theta)\, f(\theta | k \in p_i, n') d\theta \;\text{ for all } p_i \in P_{n'};$$

(ii) For every $n' \in \mathbb{N} \cup \{0\}$, the partition $P_{n'}$ is incentive compatible i.e., for any $k \in \{0, 1, ..., n'\}$ and $p_i \in P_{n'}$ such that $k \in p_i$, we have:
(4)

$$\int_0^1 U^S\left(y^{n'}_{p_i}, \theta, b\right) f\left(\theta | k, n'\right) d\theta \geq \int_0^1 U^S(y^{n'}_q, \theta, b)\, f\left(\theta | k, n'\right) d\theta, \text{ for all } q \in P_{n'}.$$

(iii) $n$ maximizes the expert's expected payoff given $\{P_{n'}\}_{n' \in \mathbb{N}\cup\{0\}}$ and $\{\mathbf{y}\left(P_{n'}\right)\}_{n' \in \mathbb{N}\cup\{0\}}$. That is, if $k \in p^{n'}(k)$, then we have:

$$(5) \qquad n \in \arg\max_{n' \in \mathbb{N}\cup\{0\}} \sum_{k=0}^{n'} \left( \int_0^1 U^S\left(y^{n'}_{p^{n'}(k)}, \theta, b\right) f\left(\theta | k, n'\right) d\theta \times \Pr\left(k | n'\right) \right) - c(n').$$

Our next step is to characterize the decision-maker's optimal action rule and the incentive compatible (IC) partitions.

LEMMA 1: *The decision-maker's sequentially rational action $y^{n'}_{p_i}$ is equal to her posterior expectation of $\theta$, given $n'$ trials and the element $p_i$ of the partition $P_{n'}$ communicated by the expert:*

$$(6) \qquad y^{n'}_{p_i} = E\left[\theta | p_i, n'\right] = \frac{1}{|p_i|} \sum_{k \in p_i} \frac{k+1}{n'+2},$$

*where $|p_i|$ denotes the cardinality of $p_i$.*

LEMMA 2: *A communication partition $P_{n'}$ is incentive compatible if and only if each element of it consists of consecutive types and the cardinalities $|p_i|$ and $|p_{i+1}|$ of any two*

[11] We do not specify explicitly which message(s) $m \in M$ signals an element $p_i$ of the partition $P_{n'}$. Any arbitrary partition of the message space $M$ into $\#P_{n'}$ sets $\mathcal{M}_1, ..., \mathcal{M}_{\#P_{n'}}$ s.t. $\cup_i \mathcal{M}_i = M$ and $\mathcal{M}_i \cap \mathcal{M}_j = \emptyset$ for $i \neq j$ will do. With any such convention, every message uniquely maps to an element of partition $P_{n'}$, for any $n'$.

*of its consecutive elements $p_i$ and $p_{i+1}$ satisfy the following:*

$$4b\left(n'+2\right)-2 \leq |p_{i+1}|-|p_i| \leq 4b\left(n'+2\right)+2. \tag{7}$$

Condition (7) is an incentive constraint for boundary types to truthfully announce the corresponding element of the partition. The first inequality guarantees that the action associated with $p_{i+1}$ is sufficiently large that the highest type in $p_i$ prefers to announce $p_i$ rather than $p_{i+1}$. The second inequality guarantees that the action associated with $p_i$ is sufficiently small that the lowest type in $p_{i+1}$ prefers to announces $p_{i+1}$ rather than $p_i$.

Note that (7) is conceptually equivalent to the arbitrage condition in Crawford and Sobel (1982), which guarantees that boundary types in an IC partition are indifferent between two consecutive elements of it. In fact, it is easy to show that as $n'$ becomes large, any IC partition of our model converges to an equilibrium partition of Crawford and Sobel (1982). The main difference is that in our model the expert is not perfectly informed, and the type space is finite. Because of the latter, our boundary types are typically not exactly indifferent between adjacent elements of the partition and also, unlike in Crawford and Sobel (1982), fully separating communication partitions can be incentive compatible. In fact, by Lemma 2 a fully separating partition is incentive compatible if and only if $b\left(n'+2\right) \leq 1/2$. If $b \geq 1/4$, then the only IC communication partition is a babbling one.

Because we will use Pareto-efficiency as a refinement criterion, let us now highlight the notions of Pareto-ranking of IC communication partitions and Pareto-efficiency of the equilibria. For any $n'$, IC partition $\{P_{n'}\}$ and sequentially rational action profile $y(P_{n'})$, the expert's and the decision-maker's ex-ante expected payoffs (derived on page 32 in the Appendix) are respectively given by:

$$E\left[-\left(y(P_{n'})-\theta\right)^2 \middle| \, P_{n'}\right]-b^2-cn' \quad \text{and} \quad E\left[-\left(y(P_{n'})-\theta\right)^2|P_{n'}\right]. \tag{8}$$

At the interim stage (i.e., after the number of trials $n'$ has been chosen but the number of successes has not yet been realized), $cn'$ is a sunk cost for the expert. So (8) implies that at the interim stage the preferences of the players are aligned: they both prefer a lower $E\left[\left(y_{p_i}^{n'}-\theta\right)^2|P_{n'}\right]$, the residual variance of $\theta$ under $\{P_{n'}\}$. Hence, all IC communication partitions for $n'$ trials can be Pareto-ranked according to the residual variance of $\theta$, or, equivalently, according to the *precision* of the decision, $1/E\left[\left(y(P_{n'})-\theta\right)^2|P_{n'}\right]$.[12]

Next, we say that equilibrium $\left(n', \{P_n\}_{n\in\mathbb{N}\cup\{0\}}, \{\mathbf{y}\left(P_n\right)\}_{n\in\mathbb{N}\cup\{0\}}\right)$ is *ex-ante Pareto efficient* if there is no other equilibrium in which the expert's and the decision-maker's ex-ante payoffs are greater, with at least one of them strictly greater. In contrast to the interim stage, at the ex-ante stage the preferences of the players are not aligned because the investment cost $cn'$ has not been incurred yet. This creates a tension between the

[12] A complete characterization of Pareto efficient IC partitions is provided in the online Appendix available at www.severinov.com/iasupplement.pdf.

common interest of the players to maximize the precision of the decision, and the fact that the cost of information acquisition is borne entirely by the expert.

Note that ex-ante Pareto efficiency requires a Pareto efficient IC partition to be used in equilibrium. However, this does not preclude the players from coordinating on a less informative communication partition off the equilibrium path, after a non-equilibrium number of trials.

### *B. The Covert Game*

In the covert game -unlike in the overt game- the decision maker does not observe the amount of information acquired by the expert. Formally, this implies that a Perfect Bayesian Equilibrium of the covert game must additionally specify the decision-maker's beliefs about the expert's information acquisition choice. The other elements are the same in both games.

We will focus on the equilibria in which the expert plays a pure strategy at the information acquisition stage. In principle, the expert may try to signal to the decision-maker information how many trials he has actually performed, via his cheap talk message. However, the next Lemma shows that restricting attention to equilibria in which the expert does not signal how much information he has acquired is without loss of generality. More precisely, it does not affect the set of equilibrium outcomes and the scope of information acquisition, which is our primary interest.

LEMMA 3: *Any outcome supported in a Perfect Bayesian Equilibrium of the covert game in which the expert follows a pure strategy in the choice of the number of trials can be supported in a Perfect Bayesian Equilibrium in which the decision-maker's beliefs about the number of trials do not vary with the expert's message.*

The intuition behind the Lemma is based on the following observation: if the expert could affect the decision-maker's beliefs about the number of performed trials, he would have a larger set of deviations available than if he could not. Specifically, if the expert could signal the number of performed trials he would have two classes of available deviations. The first one involves the expert misleading the decision-maker by performing a non-equilibrium number of trials but still sending an equilibrium message, signalling that he has performed the equilibrium number of trials. The second class of deviations involves the expert performing a non-equilibrium number of trials and signalling to the decision-maker that some (not necessarily true) non-equilibrium number of trials has been performed.

In contrast, if an expert cannot affect the decision-maker's beliefs about the number of trials, then an equilibrium has to be immune only to the deviations of the first class described above. So any equilibrium in which the expert can affect the decision-maker's beliefs about the number of trials remains an equilibrium when the expert cannot affect those beliefs.

Relying on Lemma 3, we will focus on equilibria in which, irrespectively of the expert's message, the decision-maker believes that the expert has performed the equilibrium number of trials with probability 1. Then a pure-strategy Perfect Bayesian Equilibrium of the covert game is represented by a triple $(n^*, P_{n^*}, \mathbf{y}(P_{n^*}))$, where $n^*$ is the number of trials, $P_{n^*}$ is a communication partition, and $\mathbf{y}(P_{n^*}) \equiv \{y_{p_i}^{n^*}\}_{p_i \in P_{n^*}}$ is the decision-maker's action profile. As in the overt game, the equilibrium partition $P_{n^*}$ must be incentive compatible and the action profile $\mathbf{y}(P_{n^*})$ must be sequentially rational i.e., $P_{n^*}$ and $\mathbf{y}(P_{n^*})$ have to satisfy (3) and (4), respectively.

The equilibrium number of trials must maximize the expert's expected payoff given $P_{n^*}$ and $\mathbf{y}(P_{n^*})$ i.e.,

$$(9)\quad n^* \in \arg\max_{n' \in \mathbb{N}\cup\{0\}} \sum_{k=0}^{n'} \left[ \max_{y_p \in \mathbf{y}(P_{n^*})} \int_0^1 U^S\left(y_p, \theta, b\right) f\left(\theta; k, n'\right) d\theta \right] \Pr\left(k; n'\right) - c(n').$$

The latter condition reflects the specific structure of the covert game. To understand it, consider the expected payoff that the expert gets by deviating at the information acquisition stage to some $n'$, $n' \neq n^*$. In this case, the communication game will still proceed on the basis of the equilibrium partition $P_{n^*}$ and so, whatever message the expert sends at the communication stage, he will only be able to induce one of the actions in the equilibrium action profile $\mathbf{y}(P_{n^*})$. Then, given some $k$ successes in $n'$ trials, the expert will choose to induce action $y \in \mathbf{y}(P_{n^*})$ that maximizes his payoff, as reflected in (9).

The nature of the optimality condition (9) has important implications for the covert game. In particular, the following trade-off emerges: a more informative communication partition leads to a more precise decision. However, a higher informativeness of the information partition makes it more profitable for the expert to deviate at the information acquisition stage.

The covert game, as the overt one, has multiple equilibria. We will focus on the set of Pareto-efficient ones. The definition of Pareto-efficiency of equilibria given in the previous subsection for the overt game applies to the covert game as well.

### *C. Direct Information Acquisition*

One of the central results in the literature on cheap talk is that the decisions based on information communicated by a biased expert are less precise, and hence less efficient, than the decisions made by a decision maker with direct access to the information. We inquire below whether this result continues to hold when information acquisition is costly and endogenous.

To address this question, we need to consider the benchmark problem of a decision-maker acting without an expert and acquiring information by herself. Such a decision-maker chooses a number of trials $n$ incurring the cost $c(n) = cn$. She then observes the number of successes $k \in \{0, ..., n\}$, and finally takes an action $y_{k,n}^*$. By the same argument contained in Lemma 1, the optimal action given the information acquired is: $y_{k,n}^* = E[\theta|k] = (k+1)/(n+2)$. This implies:

LEMMA 4: *The expected utility of the decision-maker who performs n trials is equal to:*

$$E\left[-\left(y^*_{k,n}-\theta\right)^2|n\right]-cn=-\frac{1}{6(n+2)}-cn. \tag{10}$$

*The decision maker's optimal number of trials $n^*(c)$ is given by:*

$$\begin{aligned} n^*(c) &= \max\left\{n:-\frac{1}{6(n+2)}-cn-\left(-\frac{1}{6(n-1+2)}-c(n-1)\right)>0\right\} \\ &= \left\lfloor\frac{1}{2}\left(\sqrt{1+\frac{2}{3c}}-3\right)\right\rfloor. \end{aligned} \tag{11}$$

Combining (10) and (11) yields a closed form expression for the decision-maker's maximal attainable expected payoff:

$$E\left[-\left(y^*_{k,n^*(c)}-\theta\right)^2|n^*\right]-cn^*(c)=-\frac{1}{6\left\lfloor\frac{1}{2}\left(\sqrt{1+\frac{2}{3c}}+1\right)\right\rfloor}-c\left\lfloor\frac{1}{2}\left(\sqrt{1+\frac{2}{3c}}-3\right)\right\rfloor. \tag{12}$$

Finally, we observe that if instead information acquisition and decision making were both delegated to a biased expert, he would maximize $E\left[-\left(y^*_{k,n}-\theta\right)^2|n\right]-b^2-cn$. It is immediate that the expert would also choose $n^*(c)$ trials.

## III. Overinvestment and Decision Precision

This section provides the main result of the paper that the decisions based on the advice of a biased expert can be more precise than the decisions based on information directly acquired by the decision maker. This is driven by a combination of two factors: the expert's overinvestment in information acquisition, and the smallness of the information loss in transmission.

To understand the intuition behind this result, note the following basic misalignment between the players' preferences. Since the cost of information acquisition is borne by the expert, ceteris paribus the decision-maker prefers that the expert acquires more information than under direct information acquisition. In contrast, the expert never wants to acquire more information than in the benchmark direct-acquisition case, and will want to acquire less information if some of it is lost in transmission.

We show that, despite the misalignment of the preferences, the principal is able to induce the expert to overinvest in information acquisition. The exact way in which this occurs is different in the overt and covert game. However, a common element in these two games is that the expert's information remains fairly coarse even under overinvestment. This helps the expert's incentives to transmit it fully, which, in turn, increases her incentives to acquire information.

It is worth noting that our overinvestment results do not rely on misalignment of preferences between the two players: they hold also when the expert is unbiased. This stands in contrast with the work Che and Kartik (2009) on the acquisition and transmission of verifiable information.

### A. *Decision Precision in the Overt Game*

The observability of the number of trials in the overt game implies that the decision-maker can and will react to the amount of information acquired by the expert. As we show below, this reaction can induce the expert to overinvest in information acquisition. We will start by considering equilibria in which the decision-maker uses the strongest credible punishment - playing a "babbling" communication equilibrium- if the expert deviates in information acquisition. We then demonstrate that such a strong threat is not necessary for overinvestment.

In a babbling equilibrium of the communication game, the decision-maker ignores the expert's message as uninformative. An intuitive interpretation of this reaction by the decision maker is that a deviation by the expert in the information acquisition stage may naturally cause the decision-maker to lose any trust in the expert.[13]

To understand how the threat of babbling can lead to an overinvestment, consider the limit case of an unbiased expert. Suppose the decision maker wants to induce him to perform $n^*(c)+1$ trials. If the expert does so, a fully separating equilibrium is played in the continuation, and the decision-maker uses the most precise action rule given $n^*(c)+1$ trials. This is incentive compatible under zero bias. If any other amount of information is acquired, a babbling equilibrium is played in the communication game: the decision-maker ignores the expert's message and takes an "uninformed" action $y = \frac{1}{2}$ equal to the ex-ante expectation of $\theta$. Thus, the expert faces a choice between two alternatives: do not perform any trials and save the cost of information acquisition, but face an "uninformed" action $y = \frac{1}{2}$; alternatively, incur the cost $(n^*(c) + 1)c$ followed by the most precise action rule. The second alternative is very close to the expert's absolute payoff maximum attained by performing $n^*(c)$ trials and fully revealing their outcome: the difference is the cost of one additional trial, $c$, which is partly compensated by higher decision precision. In contrast, the payoff difference between the first alternative and the expert's absolute payoff maximum is significant when $c$ is small and hence $n^*(c)$ is sufficiently large. So the expert prefer to overinvest and perform $n^*(c)+1$ trials. By continuity, this result also holds when the expert has a sufficiently small bias.

The next Proposition is based on this logic and identifies sufficient conditions for the existence of an equilibrium with overinvestment and a negligible loss of information in transmission. [14]

[13]Selection of a babbling equilibria to improve the decision maker's welfare is reminiscent of the constructions in the sequential cheap talk models of Aumann and Hart (2003) and Krishna and Morgan (2004). But, unlike in those constructions, we do not invoke babbling equilibria on the equilibrium path.

[14]For expositional simplicity we have assumed that the information acquisition cost is the same for the expert and the decision maker. The result of Proposition 1 holds a fortiori if the expert is more efficient than the decision maker at acquiring information.

PROPOSITION 1: *If* $b \leq \left(\sqrt{1+\frac{2}{3c}}+3\right)^{-1}$ *and* $c \leq \frac{5-\sqrt{17}}{48}$, *then the overt game has an equilibrium in which the final decision is strictly more precise than in the case of direct information acquisition by the decision maker.*

The sufficient conditions of this Proposition guarantee full information transmission on equilibrium path. However, this is not necessary for the result of the Proposition to hold. Rather what is required is that the loss of information in communication should not be too large.

To illustrate this, we have numerically computed the equilibrium of the overt game with the most precise decision rule, and compared its residual variance $E\left[(y_n-\theta)^2\,|n\right]$ with the residual variance in the benchmark direct information acquisition case in (12). We have performed these computations for $b \in [0, 0.25]$, $c \in [0, 0.027]$ and $n \leq 100$.[15] The results are presented in Figure 1(b). Figure (1a) depicts the region where the sufficient conditions of Proposition 1 hold. Taken together, these figures show that for a broad range of parameter values the precision of the decision is higher in the communication game than under direct information acquisition, even if some information is lost in communication. The overinvestment in information acquisition more than compensates for this loss.

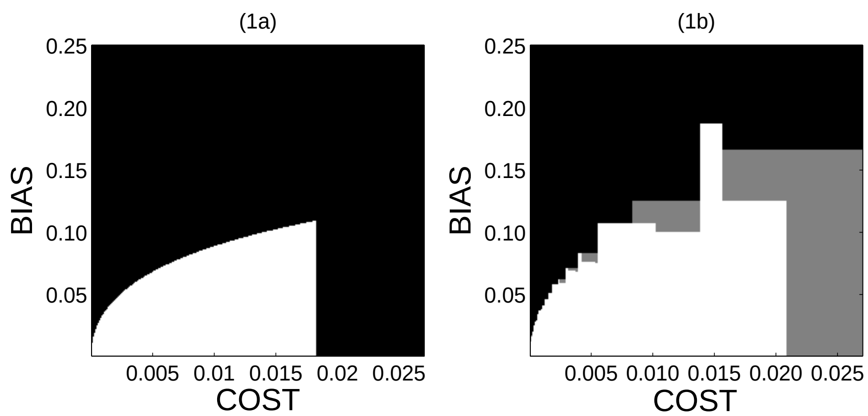


FIGURE 1. DECISION PRECISION IN THE OVERT GAME

*Note:* (1a) In the white region, the sufficient conditions in Proposition 1 are satisfied. (1b) In the white region the decision in the most informative equilibrium of the overt game is strictly more precise than with direct information acquisition. In the grey region it is as precise. In the black region it is strictly less precise.

[15] This is the relevant parameter range, since for $b \geq 0.25$ the unique equilibrium of the communication game is uninformative, and for $c > 0.02\overline{7}$ the unique solution of the decision maker's optimization problem is $n^* = 0$.

The next Proposition extends the scope of Proposition 1 by establishing that its result holds in all Pareto efficient equilibria of the communication game, except for the equilibrium preferred by the expert. In the latter, the decision precision is the same as under direct information acquisition.[16]

PROPOSITION 2: *If* $b \leq \left(\sqrt{1+\frac{2}{3c}}+1\right)^{-1}$*, then in the Pareto-efficient equilibrium of the overt game with the highest ex-ante expected payoff for the expert, the final decision has the same precision as the decision based on direct information acquisition by the decision-maker. This equilibrium can be supported when the most informative communication equilibrium is played off the path. In any other Pareto-efficient equilibrium of the overt game, the decision is strictly more precise than under direct information acquisition.*

To prove Proposition 2 we first show that full revelation of information is incentive compatible when the expert performs $n^*(c)$ trials. This implies that in the equilibrium with the highest ex-ante payoff for the expert, the latter performs exactly $n^*(c)$ trials followed by full revelation. Indeed, recall that at the ex-ante stage the players' preferences are aligned and they both would like to maximize the decision precision. So, after $n^*(c)$ trials and full revelation the expert obtains the same expected payoff as the decision-maker optimally acquiring information herself, modulo a constant $b^2$. Suppose the expert deviates and chooses a different number $n''$ of trials. Then, in any continuation equilibrium his payoff decreases by at least the same amount as the payoff of the decision maker who switches from $n^*(c)$ trials to $n''$ in direct information acquisition. Because $n^*(c)$ is optimal for the decision-maker in the latter scenario, it also constitutes an equilibrium choice for the expert.[17] This equilibrium is Pareto-efficient, because the expert attains his highest ex-ante expected payoff. By definition, in any other Pareto-efficient equilibrium the decision maker achieves a higher ex-ante expected payoff, i.e. the decision is strictly more precise.

### *B. Decision Precision in the Covert Game*

In this section we show that equilibria with overinvestment and higher decision precision also exist in the covert game, albeit under more restrictive conditions than in the overt game, because in the covert game the decision-maker does not observe the amount of information acquired by the expert and hence the latter can make unobservable deviations in the choice of the number of trials.

The logic behind this result is more subtle than in the overt game. Consider the simple case of an unbiased expert, so that full revelation is always possible on the equilibrium

[16] Focusing on Pareto efficient equilibria is standard in the literature on signaling games.

[17] Notice that this equilibrium outcome can be achieved for different selections of the communication partition to be played off-path. In particular, for the case in which the most informative communication partition is played for any number of trials acquired.

path. Relying on Lemma 3, we restrict consideration to equilibria in which the decision-maker's beliefs about the number of trials performed do not change with the expert's message. This inflexibility ultimately implies that the set of actions which the expert can induce is not tailored well to the information acquired after a non-equilibrium number of trials, and hence the decision-precision after a deviation in information acquisition is lower than on the equilibrium path. As we show below, this factor outweighs any potential expert's cost savings from a deviation to a lower number of trials, when the equilibrium number of trials does not exceed $n^*$ by too much. This logic is robust to the presence of a small bias. The following simple example illustrates this.

EXAMPLE 1: *Suppose that* $c = 1/35$ *and* $b \leq 17/210$. *By Lemma 4,* $n^* = 0$ *i.e., the decision maker would not acquire any information, receiving a payoff of* $-1/12$ *. However, the covert game has an equilibrium in which the expert performs one trial and reveals its outcome, inducing action* $y = 1/3$ *after a failure and* $y = 2/3$ *after a success. The associated expected payoffs of the expert and of the decision maker are* $-1/18 - b^2 - c$ *and* $-1/18$, *respectively. So, the decision precision is higher than under direct information acquisition, and the decision-maker's expected payoff increases by 50 percent. Let us check that the expert has no profitable deviations. After any deviation, he can only induce one of the equilibrium actions,* $y = 1/3$ *or* $y = 2/3$. *If he deviates to zero trials, then because of his upwards bias* $b > 0$, *he would induce* $y = 2/3$ *obtaining expected utility of* $-1/9 + b/3 - b^2$. *This is less than* $-1/18 - b^2 - c$ *when* $b \leq 17/210$ *and* $c = 1/35$, *so this deviation is unprofitable. Showing that a deviation to* $n > 1$ *is unprofitable is straightforward and is omitted.*

Example 1 deals with the simplest case in which the only downward deviation in information acquisition involves performing no trials. But our line of argument works more generally. Indeed, in Example 2 below the decision-maker acquiring information directly performs one trial, but in an equilibrium of the covert game the expert performs two trials. In this case a downward deviation by an expert to a single trial still generates a non-trivial, binary information partition, while the equilibrium action profile consists of three elements and hence offers a finer choice to the agent. Yet, this action profile is not well-suited to the off-equilibrium information partition, and a deviation to one trial causes a loss of decision precision which is not compensated by an economy of information acquisition cost.

EXAMPLE 2: *Suppose that* $b \leq 1/24$ *and* $1/72 < c < 1/48$. *By Lemma 4, the decision-maker would acquire one trial and get a payoff* $-1/18 - c$ *. However, the covert game has an equilibrium in which the expert performs two trials and truthfully reveals the outcome, inducing actions* $1/4$, $1/2$, *and* $3/4$ *after zero, one, and two successes, respectively. The expected payoffs of the expert and of the decision maker are* $-1/24 - b^2 - 2c$ *and* $-1/24$, *respectively, with the utility gain to the decision-maker between 40 percent and 45 percent depending on the cost.*

*By Lemma 2 truthful revelation of the trial outcomes is incentive compatible for the expert. Let us check that there are no profitable deviations at the information acquisition stage. Any message after such deviation can only induce one of the equilibrium actions,*

$1/4$, $1/2$, *or* $3/4$. *If the expert deviates to zero trials, then her payoffs from these actions are* $-1/12-(b+1/4)^2$, $-\frac{1}{12}-b^2$, *and* $-\frac{1}{12}-b^2$, *respectively. Since* $b^2 \leq \frac{1}{24}$, *action* $\frac{1}{2}$ *gives the highest payoff, which is nevertheless smaller than his putative equilibrium payoff* $-1/24-b^2-2c$ *because* $c < 1/48$. *So this deviation is unprofitable. Next, consider a deviation to* $n=1$. *In this case, the number of successes k could be either 0 or 1. The computations provided in the Appendix show that the expert prefers to action* $1/4$ *when* $k=0$, *and action* $3/4$ *when* $k=1$. *So, after* $n=1$ *the expert's expected payoff equals* $Prob(k=0|n=1)\left(-2\int_0^1\left(\frac{1}{4}-\theta-b\right)^2(1-\theta)d\theta\right)+Prob(k=1|n=1)\left(-2\int_0^1\left(\frac{3}{4}-\theta-b\right)^2\theta d\theta\right)-c$ $=-3/48-b^2-c$ *which is less than his putative equilibrium payoff* $-1/24-b^2-2c$. *Finally, showing that the expert would not deviate to* $n>2$ *is straightforward and is therefore omitted.*

The following Proposition generalizes the above examples and identifies sufficient conditions for strict overinvestment and higher decision precision in the communication game.

PROPOSITION 3: *If* $\frac{1}{6(n+2)(n+3)} < c < \frac{1}{6(n+1)(n+3)} - \max\left\{0, \left(\frac{1}{3}b\right)\mathbb{I}_{n=0}, \left(\frac{24b-1}{96}\right)\mathbb{I}_{n=1}, \left(\frac{30b-1}{450}\right)\mathbb{I}_{n=2}, \left(\frac{30b-1}{360}\right)\mathbb{I}_{n=3}, \left(\frac{63b-2}{735}\right)\mathbb{I}_{n=4}\right\}$ *for some integer n*[18], *and* $b \leq \frac{1}{4(n+3)}$, *then the covert game has an equilibrium in which the final decision is strictly more precise than the decision based on direct information acquisition.*

The conditions of Proposition 3 are represented graphically in Figure 2a. Observe that an interval of costs for which these conditions hold, provided that the bias $b$ is sufficiently small, is followed by an interval of slightly higher costs for which these conditions never hold, which in turn is followed by an interval of higher costs for which these condition holds again under small bias, and so on. This pattern reflects the following regularity.

Let $H(n)$ be an interval of cost values for which $n$ is the optimal number of trials under direct information acquisition. Note that $H(n)=\left(\frac{1}{6(n+2)(n+3)},\frac{1}{6(n+1)(n+2)}\right)$, and so the cost axis can be divided into adjacent intervals $H(n)$ corresponding to different values of $n$. For each $n$, the second condition in Proposition 3 identifies a subinterval $L(n)$ of $H(n)$ where the result holds. $L(n)$ constitutes the lower part of $H(n)$ for every $n$. Hence, the intervals $L(n)$ are not adjacent.

When the unit cost $c$ lies in $L(n)$ and the bias is not too large, the covert communication game admits an equilibrium in which the expert runs $n+1$ trials and fully reveals their outcome. $L(n)$ is a strict subset of $H(n)$ because, if $c$ is too close to the upper bound of $H(n)$, the expert prefers to save some cost and deviate to $n$ trials. The condition on the bias i.e., $b \leq 1/[4(n+3)]$, guarantees that, if the expert performs $n+1$ trials, he then fully reveals their realization in the communication game.

[18]The symbol $I_{n=k}$ in the inequality denotes the indicator function taking value one if $n=k$, and zero otherwise. It is easy to check that this interval is non-empty for every $n$.

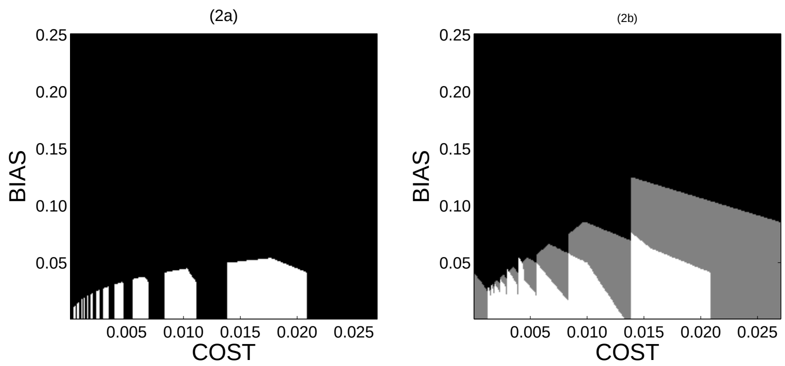


FIGURE 2. DECISION PRECISION IN THE COVERT GAME

*Note:* (2a) In the white region, the sufficient conditions in Proposition 3 hold. (2b) In the white region, the decision in the most informative equilibrium of the covert game is strictly more precise than under optimal direct information acquisition. In the grey region the precision is the same in the most informative equilibrium of the covert game and under optimal direct information acquisition. In the black region, the decision under optimal direct information is more precise than in the most informative equilibrium of the covert game.

Since the conditions of Proposition 3 are stronger than necessary, we have numerically identified the whole region of the parameter space where the precision of the decision maker's action in the most informative equilibrium of the covert game is strictly higher than under optimal direct information acquisition. The results are presented in Figure 2b.

We conclude the analysis of the covert game with a result analogous to Proposition 2 for the overt game.

PROPOSITION 4: *If* $b \leq \left(2\sqrt{1+\frac{2}{3c}}+2\right)^{-1}$, *then in the Pareto-efficient equilibrium of the covert game with the highest ex-ante expected payoff for the expert, the final decision has the same precision as the decision based on direct information acquisition. In every other Pareto efficient equilibrium of the covert game the decision is strictly more precise.*

The proof of Proposition 4 establishes that in his preferred equilibrium the expert performs $n^*(c)$ trials and fully reveals their outcome. The key step of the proof shows that the expert cannot benefit by deviating from $n^*$ at the information acquisition stage, because any such deviation yields him a loss exceeding the loss incurred by the decision-maker making the same deviation in direct information acquisition. The second part of the Proposition follows because in any other Pareto efficient equilibrium the decision-maker's payoff, and hence the precision of the decision, must be higher.

## IV. Organization Design

Our results have important implications for organization design. Specifically, consider an organization that has to gather information and take a decision under uncertainty. We will focus on two cases which differ in terms of organizational objectives, depending on whose interests are served by the organization. The first case deals with an organization that serves primarily the interests of its principal. Here, we consider three possible organizational structures. Under *centralization*, the principal acquires information directly and takes the decision. Formally, it corresponds to our benchmark case of direct information acquisition by the decision-maker. Beyond a literal interpretation, we may consider this organizational structure to describe situations in which the principal hires, directs and closely supervises its employees. Under *delegation*, the principal delegates both information acquisition and decision-making to an agent. Finally, in a *communication-based organization*, the principal delegates the task of information acquisition to her agent but keeps the decision-making authority.

Under centralization/direct information acquisition, the principal's expected payoff is given by the following:

$$E\left[-\left(y^{*}-\theta\right)^{2}|n^{*}(c)\right]-cn^{*}(c), \tag{13}$$

where $y_k^{n^*} = (k+1)\,/\,[n^*\,(c)+2]$ is the principal's optimal decision rule where $k$ is the number of successes observed in $n^*\,(c)$ trials.

The optimization problem solved by the agent under delegation is similar to the principal's problem under centralization. In both cases, the party acquiring information optimally conducts $n^*(c)$ trials given by (11). But under delegation, the optimal decision rule for the agent is $y_k^{n^*}(b) = (k+1)\,/\,[n^*\,(c)+2]+b$. So, the principal's expected payoff under delegation is:

$$E\left[-\left(y^{*}-\theta\right)^{2}|n^{*}\,(c)\right]-b^{2}. \tag{14}$$

The comparison between centralization and delegation is straightforward. By delegating decision to the agent, the principal trades off saving the information acquisition cost $cn^*(c)$, for a biased decision with loss $b^2$.

Communication can be modelled either via overt or covert game analyzed above, depending on whether the principal observes the number of trials performed by the agent or not. In either game, the principal's expected payoff is determined by the number $n$ of trials and equilibrium communication partition $\mathcal{P}$, and is equal to:

$$E\left[-\left(\bar{y}-\theta\right)^{2}|\mathcal{P}\right], \tag{15}$$

where $\bar{y} = E(\theta|p^i)$ is the optimal decision when the expert's message signals element

$p^i$ of the partition $\mathcal{P}$.

Inspection of expressions (13)-(15) shows that both centralization and delegation are dominated by a communication equilibrium with partition $\mathcal{P}$ if $E\left[-(\bar{y}-\theta)^2|\mathcal{P}\right] \geq E\left[-(y^*-\theta)^2|n^*(c)\right]$, or, in our terminology, if the decision in the communication equilibrium is at least as precise as in the benchmark case of direct information acquisition. Hence, Propositions 2 and 4 imply the following Corollary.

COROLLARY 1: ***(a)*** *If* $b \leq \left(\sqrt{1+\frac{2}{3c}}+1\right)^{-1}$*, then the principal strictly prefers any Pareto efficient equilibrium of the overt game to centralization and delegation.*

***(b)*** *If* $b \leq \left(2\sqrt{1+\frac{2}{3c}}+2\right)^{-1}$*, then the principal strictly prefers any Pareto efficient equilibrium of the covert game to centralization and delegation.*

The comparison between communication and centralization in Corollary 1 is performed under the assumption that the agent and the principal have the same cost $c$ in information acquisition. However, the comparison between communication and delegation does not rely on it, since the expert bears the cost of information acquisition in both organizations.

Next, to highlight the benefit that the principal gets from communication-based organization, we have computed and plotted in Figure 3 the change in the decision-maker's expected payoff obtained by switching from direct information acquisition to a communication-based organization, in an equilibrium with the minimal overinvestment. The former payoff is given by equation (12), while the latter corresponds to the communication equilibrium in which the expert performs $n^*(c)+1$ trials and reveals their outcomes (Sufficient conditions for the existence of this equilibrium in the overt and covert game are provided in Propositions 1 and 3). The actual gain for the decision-maker is larger than the one represented on the graph in equilibria larger overinvestment[19].

The lower curve on the graph represents the gain from higher precision of the decision. The latter varies from a few percentage points for low values of the cost $c$, to around 25 percent when $c$ is close to the top of the admissible range. The higher curve on the graph shows that the gain is much larger, and varies from 40 percent to 52 percent, when the beneficial effect of transferring the cost from the decision-maker under direct information acquisition to the expert under communication is also accounted for.

As our second application, we consider an organization whose objective function is the sum of payoffs of both parties who are affected by the decision. While we continue to refer to them as principal and agent for ease of comparison, in the present case it is more natural to think about the organization as a partnership. Thus, the total payoff of the partnership is given by $2E\left[-(\bar{y}(\mathcal{P})-\theta)^2|\mathcal{P}\right]-b^2-cn$, where $\mathcal{P}$ is incentive compatible information partition, $\bar{y}(\mathcal{P})$ is the action profile under $\mathcal{P}$, and $n$ is the number of trials performed and $b$ is bias which in this case reflects the non-congruence of interests between the partners. Note that a non-congruence of interests between partners is a

[19] We have plotted the two curves in Figure 3 for the same range of costs as Figures. As stated in Propositions 1 and 3, an equilibrium with of the overt and covert game respectively, in which the expert performs $n^*(c)+1$ trials and reveals their outcomes, exists only in subsets of this range. These subsets are illustrated in Figures 1(a) and 2(a), respectively.

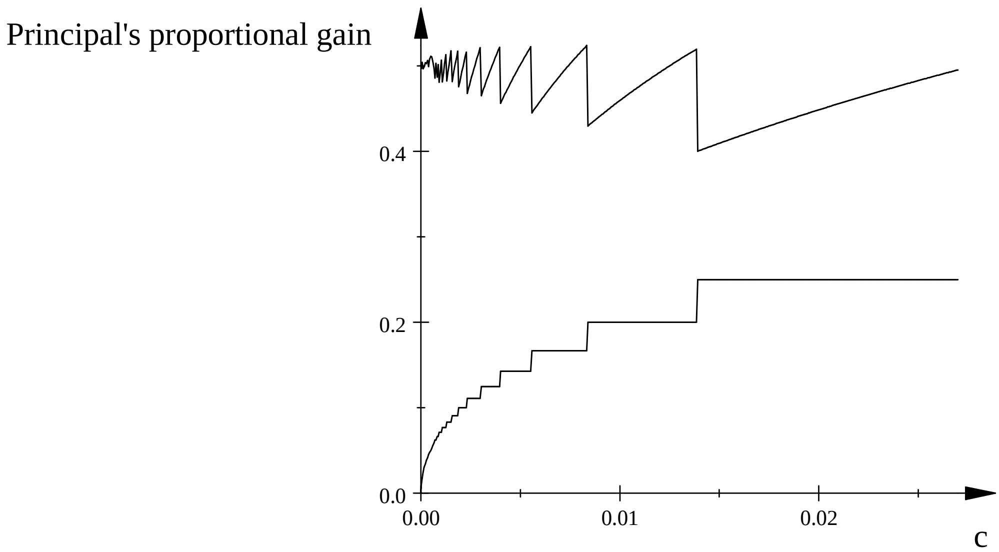


FIGURE 3. PRINCIPAL'S GAIN

*Note:* The lower line represents the proportional increase in the decision precision when switching from $n^*(c)$ trials directly purchased by the principal to $n^*(c)+1$ trials purchased by the expert and followed by full revelation of their outcome. The higher line represents the proportional increase in the principal's payoff when taking into account both the increase in the decision precision and the cost savings.

common phenomenon (multiple lawsuits between partners attest to that), so the bias $b$ and the fact that the partner who chooses the action maximizes her own payoff rather than the total payoff above are a natural reflection of such misalignment.

Although a partnership can be organized in many different ways, our analysis will be limited to the organizational forms described above. Thus, *under centralization* one partner takes upon himself both information acquisition and decision-making. Formally, under partnership's objective function, centralization is payoff-equivalent, to both our benchmark case of direct information acquisition by the decision -maker and to the delegation. Under *specialization or division of labor,* the tasks of information acquisition and decision making are split between the partners. This organizational form can be modelled either via the overt or the covert game, depending on whether the decision-maker observes the information acquisition choice by the other partner or not.

Our next result shoes that, under the sufficient conditions of Propositions 1 and 3, the outcome of specialization/division of labor (i.e. at least one equilibrium outcome of both the overt and covert game) dominates centralization.

PROPOSITION 5: ***(a)*** *If* $b \leq \left(\sqrt{1+\frac{2}{3c}}+3\right)^{-1}$ *and* $c \leq \frac{5-\sqrt{17}}{48}$*, division of labor in the overt game yields a higher payoff to the partnership than centralization in at least one equilibrium.*
***(b)*** *For any integer n, if* $b \leq \frac{1}{4(n+3)}$*, and*
$\frac{1}{6(n+2)(n+3)} < c < \frac{1}{6(n+1)(n+3)} - \max\left\{0, \left(\frac{1}{3}b\right)\mathbb{I}_{n=0}, \left(\frac{24b-1}{96}\right)\mathbb{I}_{n=1}, \left(\frac{30b-1}{450}\right)\mathbb{I}_{n=2}, \left(\frac{30b-1}{360}\right)\mathbb{I}_{n=3}, \left(\frac{63b-2}{735}\right)\mathbb{I}_{n=4}\right\}$*, division of labor in the covert game yields a higher payoff to the partnership than centralization in at least one equilibrium.*

Since ours is not a complete study of partnership mechanisms, we do not claim to provide a characterization of the optimal organizational form for a partnership. However, the last result suggest that a partnership involved in information acquisition and decision-making in an uncertain environment would benefit from specialization and division of labor between the partners. The equilibria identified in Propositions 1 and 3 not only yield a significant utility gain to the decision-maker, but also generate a surplus that would be sufficient to compensate the acquirer of information for the cost and effort that he expends in this activity.

## V. Conclusions

We have developed a simple, yet intuitive model of costly endogenous information acquisition with strategic communication of this information. In this context, we have shown that decisions based on a biased expert's advice may be more precise than optimal choices based on direct information acquisition, even if the expert is not more efficient than the decision maker at acquiring information. This result is important for organization design, as it implies that (i) under certain conditions communication-based organizations outperform delegation and centralization, and (ii) under certain conditions partnerships are better off dividing the information acquisition and decision making among the partners, rather than centralizing these tasks to a single partner. In this respect, our paper contributes to the literature that employs a strategic communication framework to study optimal allocation of authority in the presence of incomplete information.

We have derived our results for a specific information acquisition model, but we would like to highlight that the main forces behind our results are robust to more general statistical structures. In the overt game, the use of a credible threat of the worst off-path punishment to induce overinvestment would also be effective in incentivizing the expert in different settings with either continuous or discrete information. As for the covert game, consider any communication model in which the sender's information is fixed. We know from Crawford and Sobel (1982) that unless the sender is unbiased, the set of messages used on the equilibrium path is discrete (up to outcome equivalence), and depends on the amount of information held by the expert. When considering covert information acquisition, an expert deviating from the equilibrium information acquisition choice would be penalized by the inflexibility of equilibrium language. However, to conclude that overinvestment is beneficial for the decision maker, it would need to be the case that (i) the fixed language effect is sufficiently strong relative to the information acquisition cost to deter deviation at the information acquisition stage and (ii) that the information transmission loss is sufficiently small not to offset the equilibrium overinvestment[20]. We leave this issue for further research.

A number of other interesting questions can be addressed in the framework of our model. First, suppose that the decision-maker was able to subsidize the expert's information acquisition cost. How would that affect the amount of information acquired and the precision of the decision? Second, how would the outcome of the communication game be affected if the expert acquired the information covertly but had an option to verifiably disclose the amount of information that he acquired? Would a decision maker prefer knowing the amount of information acquired by an expert, when she could not inspect its content? As shown by Austen-Smith (1994), this issue is far from being transparent. We leave these and other questions for future research.

## Mathematical Appendix

**Proof of Lemma** 1**.** The decision maker chooses $y_{p_i}^{n'}$ so as to maximize

$$-\int_0^1 \left(y_{p_i}^{n'} - \theta\right)^2 f\left(\theta | k \in p_i, n'\right) d\theta.$$

Taking the first-order condition, we obtain $y_{p_i}^{n'} = \int_0^1 \theta f\left(\theta | k \in p_i, n'\right) d\theta = E\left[\theta | p_i, n'\right]$. Simplifying:

$$E\left[\theta | p_i, n'\right] = E\left[E\left[\theta | k, n'\right] | k \in p_i\right] = \sum_{k \in p_i} E\left[\theta | k, n'\right] \frac{f\left(k; n'\right)}{\sum_{k \in p_i} f\left(k; n'\right)} = \frac{1}{|p_i|} \sum_{k \in p_i} \frac{k+1}{n'+2}$$

because $E\left[\theta | k, n'\right] = \frac{k+1}{n'+2}$, and $f\left(k; n'\right) = \int_0^1 f\left(k; n', \theta\right) d\theta = \dfrac{n'!}{k!\,(n'-k)!} \int_0^1 \theta^k (1-\theta)^{n'-k} d\theta = \dfrac{n'!}{k!\,(n'-k)!} \frac{k!\left(n'-k\right)!}{(n'+1)!} = \frac{1}{n'+1}$.

**Proof of Lemma** 2. First, we show that the incentive compatibility constraint (4) can be rewritten as

$$-\left(y_{p_i}^{n'} - y_q^{n'}\right)\left[\left(y_{p_i}^{n'} + y_q^{n'}\right) - 2E\left[\theta / k, n'\right] - 2b\right] \geq 0 \textit{ for all } q \in P_{n'}.$$

[20] We would like to thank an anonymous referee for this observation.

For this, note the following:

$$\int_0^1 U^S\left(y_{p_i}^{n'},\theta,b\right) f\left(\theta;k,n'\right)d\theta \geq \int_0^1 U^S(y_q^{n'},\theta,b) f\left(\theta;k,n'\right)d\theta$$

$$-\int_0^1 \left[\left(y_{p_i}^{n'}-\theta-b\right)^2-\left(y_q^{n'}-\theta-b\right)^2\right] f\left(\theta;k,n'\right)d\theta \geq 0$$

$$-\int_0^1 \left[\left(y_{p_i}^{n'}\right)^2+(\theta+b)^2-2y_{p_i}^{n'}(\theta+b)-\left(y_q^{n'}\right)^2-(\theta+b)^2+2y_q^{n'}(\theta+b)\right] f\left(\theta;k,n'\right)d\theta \geq 0$$

$$-\int_0^1 \left[\left(y_{p_i}^{n'}\right)^2-\left(y_q^{n'}\right)^2-2\left(y_{p_i}^{n'}-y_q^{n'}\right)(\theta+b)\right] f\left(\theta;k,n'\right)d\theta \geq 0$$

$$-\left(y_{p_i}^{n'}-y_q^{n'}\right)\left[\left(y_{p_i}^{n'}+y_q^{n'}\right)-2E\left[\theta/k,n'\right]-2b\right] \geq 0$$

Next, we prove that in any pure-strategy equilibrium of the communication subgame, each element of the equilibrium partition is connected. Suppose by contradiction that there exists an equilibrium where at least one element of the partition is not connected. Then, there exists at least a triple of types $\left(k,k',k''\right)$ such that: $k<k''<k'$, $k$ and $k'$ belong to the same element of the partition, which we denote by $p_a$, and $k''$ belongs to a different element, which we denote by $p_b$. Let $y_a$ and $y_b$ be the equilibrium actions associated to $p_a$ and $p_b$ respectively. By incentive compatibility, the following inequalities must hold:

$$\begin{aligned}
(y_b-y_a)\left(y_a+y_b-\frac{2(k+1)}{n'+2}-2b\right) &\geq 0\\
(y_b-y_a)\left(y_a+y_b-\frac{2\left(k'+1\right)}{n'+2}-2b\right) &\geq 0\\
(y_a-y_b)\left(y_a+y_b-\frac{2\left(k''+1\right)}{n'+2}-2b\right) &\geq 0
\end{aligned}$$

Because the first two expressions are positive, then $y_a+y_b-\frac{2(k+1)}{n+2}-2b$ and $y_a+y_b-\frac{2\left(k'+1\right)}{n+2}-2b$ have the same sign. But then, also $y_a+y_b-\frac{2\left(k''+1\right)}{n+2}-2b$ has the same sign, because $k<k''<k$. And hence, the last expression is negative: A contradiction.

Next, we prove that incentive compatibility implies expression (7). Let $k$ be the expert's type. Denote by $y$ the equilibrium action associated to $k$, and by $\widetilde{y}$ any other equilibrium action. The incentive compatibility constraint is:

$$(\widetilde{y}-y)\left(\widetilde{y}+y-\frac{2(k+1)}{n'+2}-2b\right)\geq 0. \tag{A1}$$

First, we rule out the possibility that a type $k$ deviates by inducing an equilibrium action

$\widetilde{y}$ larger than $y$. This deviation is unprofitable if and only if

$$\widetilde{y}+y-\frac{2(k+1)}{n'+2}-2b\geq 0. \tag{A2}$$

Because the expression is increasing in $\widetilde{y}$ and decreasing in $k$, it immediately follows that the tightest incentive compatibility constraints concern the highest type $\overline{k}$ in any element $p_i$ of the equilibrium partition, entertaining the possibility of deviating and inducing the equilibrium action $\widetilde{y}$ associated to $p_{i+1}$, the element of the partition immediately to the right of $p$.

Hence, we now consider such constraints. The explicit expression for $y$ and $\tilde{y}$ are:

$$\begin{aligned} y &= \frac{1}{|p_i|}\left[\frac{\overline{k}+1}{n'+2}+\frac{\overline{k}-1+1}{n'+2}+...+\frac{\overline{k}-(|p_i|-1)+1}{n'+2}\right]=\frac{2\overline{k}-|p_i|+3}{2(n'+2)} \\ \widetilde{y} &= \frac{1}{|p_{i+1}|}\left[\frac{\overline{k}+1+1}{n'+2}+\frac{\overline{k}+2+1}{n'+2}+...+\frac{\overline{k}+|p_{i+1}|+1}{n'+2}\right]=\frac{2\overline{k}+|p_{i+1}|+3}{2(n'+2)} \end{aligned}$$

Hence, condition (A2) simplifies as:

$$\frac{2\overline{k}+|p_{i+1}|+3}{2(n'+2)}+\frac{2\overline{k}-|p_i|+3}{2(n'+2)}-\frac{2\left(\overline{k}+1\right)}{n'+2}-2b\geq 0,$$

or,

$$|p_{i+1}|\geq |p_i|+4b(n+2)-2. \tag{A3}$$

Proceeding in the same fashion, we prove that when $\widetilde{y}<y$, the tightest incentive compatibility constraints concern the lowest type $\underline{k}$ in any element $p_i$ of the equilibrium partition, entertaining the possibility of deviating and inducing the equilibrium action $\widetilde{y}$ associated to $p_{i-1}$, the element of the partition immediately to the left of $p_i$. Again, letting $j$ be the cardinality of $p_i$, and $z$ be the cardinality of $p_{i-1}$, we obtain

$$y=\frac{1}{|p_i|}\left[\frac{\underline{k}+1}{n'+2}+\frac{\underline{k}+1+1}{n'+2}+...+\frac{\underline{k}+|p_i|-1+1}{n'+2}\right]=\frac{2\underline{k}+|p_i|+1}{2(n'+2)}$$

$$\widetilde{y} = \frac{1}{|p_{i-1}|}\left[\frac{\underline{k}-1+1}{n'+2}+\frac{\underline{k}-2+1}{n'+2}+...+\frac{\underline{k}-|p_{i-1}|+1}{n'+2}\right] = \frac{2\underline{k}-|p_{i-1}|+1}{2(n'+2)}$$

Hence, condition (A2) simplifies as:

$$\frac{2\underline{k}-|p_{i-1}|+1}{2(n'+2)}+\frac{2\underline{k}+|p_i|+1}{2(n'+2)}-\frac{2\left(\underline{k}+1\right)}{n'+2}-2b\leq 0$$

which implies

$$|p_i| \leq |p_{i-1}| + 4b(n+2)+2. \tag{A4}$$

**Derivation of Expression (8).** For any $n'$, consider the expert's and the decision-maker's expected payoffs associated to IC partition $\{P_{n'}\}$, assuming that the decision-maker plays her sequentially rational strategy, as described by Lemma 1:

$$\sum_{k=0}^{n'}\left(\int_0^1 U^S\left(y^{n'}_{p^{n'}(k)},\theta,b\right) f\left(\theta;k,n'\right)d\theta\times\Pr\left(k;n'\right)\right)-c(n') \tag{A5}$$

$$\sum_{k=0}^{n'}\left(\int_0^1 U^R\left(y^{n'}_{p^{n'}(k)},\theta,b\right) f\left(\theta;k,n'\right)d\theta\times\Pr\left(k;n'\right)\right) \tag{A6}$$

Let the operator $E[.|P_{n'}]$ denote the expectation with respect to $\theta$ and $k$ conditional on the number of experiments $n'$, and the associated partition $P_{n'}$. Then, using the fact that, by (6), $E\left[y(P_{n'})|P_{n'}\right] = E[\theta|P_{n'}]$, we can rewrite the expert's expected payoff in (A5) as follows:

$$\begin{aligned}E\left[-\left(y(P_{n'})-\theta-b\right)^2|P_{n'}\right]-cn' &= E\left[-\left(y(P_{n'})-\theta\right)^2+2b\left(y(P_{n'})-\theta\right)|P_{n'}\right]-b^2-cn'\\ &= E\left[-\left(y(P_{n'})-\theta\right)^2\middle| P_{n'}\right]-b^2-cn',\end{aligned}$$

Further, the decision-maker's expected payoffs in (A6) can be rewritten as:

$$E\left[-\left(y(P_{n'})-\theta\right)^2|P_{n'}\right]$$

**Proof of Lemma 3:** Consider an equilibrium $\mathcal{E}^1 = (n^1, m^1(n,k), B^1(.), \sigma^1)$ in which the expert performs $n^1$ trials, and follows message strategy $m^1(n,k)$, where $n$ is the number of trials and $k$ is the number of successes, the decision-maker forms beliefs $B^1(.) : \mathcal{M} \mapsto \Delta(\{(n,k)|n,k \in \mathcal{N}, n \geq k\})$ and follows action-choice strategy $\sigma^1(.) : B^1 \mapsto \Delta([0,1])$. [21] Note that the decision maker's beliefs $B^1(.)$ is a mapping from

[21] In this proof, we need to use a more canonical definition of perfect Bayesian equilibrium, not relying on partitions.

the set of expert's messages $\mathcal{M}$ into the set of probability distributions $\Delta(\{(n,k)|n,k \in \mathcal{N}, n \geq k\})$, reflecting the fact that in the covert game the decision maker has to form beliefs not only about the number of successes but also about the number of experiments performed by the expert.

Let $\mathcal{M}^e = \{m^1(n^1,k)|k = 0, 1, ..., n^1\}$ be the set of messages sent on the equilibrium path with a positive probability. Then $B^1_{|N}(m)$ puts probability 1 on $n^1$ for all $m \in \mathcal{M}^e$. Next, fix some arbitrary $\breve{m} \in \mathcal{M}^e$ and consider modified belief $\hat{B}(.)$ and modified strategy $\hat{\sigma}(.)$ such that for any $m \in \mathcal{M}^e$, $\hat{B}(m) = B^1(m)$ and $\hat{\sigma}(m) = \sigma^1(m)$, while for any $m \in \mathcal{M} \setminus \mathcal{M}^e$, $\hat{B}(m) = B^1(\breve{m})$ and $\hat{\sigma}(m) = \sigma^1(\breve{m})$. Hence, $\hat{B}(.)$ puts probability 1 on $n^1$ for all $m \in \mathcal{M}$.

Now consider a putative equilibrium $\hat{\mathcal{E}} = (n^1, m^1(n,k), \hat{B}(.), \hat{\sigma}(.))$ in which the expert performs $n^1$ trials and follows message strategy $m^1(n,k)$, and the decision-maker uses belief rule $\hat{B}(.)$ and strategy profile $\hat{\sigma}(.)$. With the decision-maker's belief rule $\hat{B}(.)$ in $\hat{\mathcal{E}}$, no expert's message can change the decision-maker's beliefs about the number of trials.

Furthermore, $\hat{\mathcal{E}}$ does constitute a perfect Bayesian equilibrium because $\mathcal{E}^1$ is a perfect Bayesian equilibrium, and both $\hat{\mathcal{E}}$ and $\mathcal{E}^1$ prescribe the same behavior and beliefs on the equilibrium path, with the only difference between them being in the beliefs off the equilibrium path i.e., after a message $m \in \mathcal{M} \setminus \mathcal{M}^e$: after such message $\hat{\mathcal{E}}$ prescribes beliefs $\hat{B}(m) = B^1(\breve{m})$, while $\mathcal{E}^1$ prescribes beliefs $B^1(m)$. However, since a message $\breve{m}$ is also available to a deviating expert in $\mathcal{E}^1$ but does not lead to a profitable deviation, there is no profitable deviation for an expert in $\hat{\mathcal{E}}$. So, $\hat{\mathcal{E}}$ is a perfect Bayesian equilibrium. ■

**Proof of Lemma 4**: First, recall that $y^*_k = E[\theta|k] = (k+1)/(n+2)$. Using this expression below we obtain:

$$E\left[-\left(y^*_k-\theta\right)^2|n\right]-cn = -\sum_{k=0}^{n}\Pr(k;n)\int_0^1 \left(E[\theta|k]-\theta\right)^2 f(\theta;k,n)-cn$$

$$\begin{aligned}
&= -\sum_{k=0}^{n}\frac{1}{n+1}\int_0^1\left(\frac{k+1}{n+2}-\theta\right)^2\frac{(n+1)!}{k!\,(n-k)!}\theta^k(1-\theta)^{n-k}\,d\theta-cn\\
&= -\sum_{k=0}^{n}\frac{1}{n+1}\int_0^1\left[\left(\frac{k+1}{n+2}\right)^2+\theta^2-2\theta\left(\frac{k+1}{n+2}\right)\right]\frac{(n+1)!}{k!\,(n-k)!}\theta^k(1-\theta)^{n-k}\,d\theta-cn\\
&= -\sum_{k=0}^{n}\frac{1}{n+1}\left[\int_0^1\theta^2\frac{(n+1)!}{k!\,(n-k)!}\theta^k(1-\theta)^{n-k}\,d\theta-\left(\frac{k+1}{n+2}\right)^2\right]-cn\\
&= -\sum_{k=0}^{n}\frac{1}{n+1}\left[\frac{(k+2)(k+1)}{(n+3)(n+2)}-\left(\frac{k+1}{n+2}\right)^2\right]-cn\\
&= -\frac{1}{6(n+2)}-cn. \qquad ■
\end{aligned}$$

**Proof of Proposition 1.** We prove that there exists an equilibrium of the overt information acquisition game in which the expert runs $n^*(c)+1$ trials and fully reveals their realizations. Clearly, this equilibrium implies a decision precision higher than the benchmark of direct information acquisition by the decision maker. The result then follows because either this equilibrium is Pareto-efficient, or there exists another equilibrium which Pareto-dominates it, in which the payoff of the decision-maker, i.e. the decision precision, is even higher.

The proof proceeds as follows. First, we find the maximal number of trials $\widetilde{n}(c)$ such that, under a given investment cost $c$, the utility that the expert obtains by conducting $\widetilde{n}(c)$ trials and fully revealing their realizations to the decision maker is higher than the utility from running any other number of trials and playing the babbling equilibrium. Formally, $\widetilde{n}(c)$ is the highest integer that satisfies

$$-\frac{1}{6(n+2)}-b^2-cn \geq -\frac{1}{12}-b^2.$$

Further, from Lemma 2 it follows that $\widehat{n}(b) \equiv \left\lfloor \frac{1}{2b}-2 \right\rfloor$ is the maximal number of trials for which full revelation in the communication game is incentive compatible. Hence, it is an equilibrium for the expert to run $n^*(c)+1$ trials and to fully reveal the information to the decision maker whenever the following condition holds:

$$n^*(c)+1 \leq \max\{\widehat{n}(b), \widetilde{n}(c)\}. \tag{A7}$$

The condition $n^*(c)+1 \leq \widetilde{n}(c)$ is satisfied if $\sqrt{\frac{2+3c}{12c}}-\frac{3}{2}+1 \leq \frac{1}{12c}-2$, i.e., $c \leq \frac{5-\sqrt{17}}{48}$, whereas the condition $n^*(c)+1 \leq \widehat{n}(b)$ is satisfied if $\sqrt{\frac{2+3c}{12c}}-\frac{3}{2}+1 \leq \frac{1}{2b}-2$, or $b \leq \left(\sqrt{1+\frac{2}{3c}}+3\right)^{-1}$.

If $\widehat{n}(b) \geq n^*(c)+1$ and $\widetilde{n}(c) \geq n^*(c)+1$, then there exists an equilibrium of the overt information acquisition game in which the expert runs $n^*(c)+1$ trials and fully reveals their realizations, while the babbling equilibrium is played in any subgame in which $n' \neq n$ trials are run. The decision maker's utility $E\left[-\left(y_p-\theta-b\right)^2 | P_n\right]$ in this equilibrium is $-1/[6(n^*+1+2)]$ which is strictly larger than the decision maker's utility $-1/[6(n^*+2)]$ if she directly acquired information. Q.E.D.

**Proof of Proposition 2.** We start by proving that under the condition of the theorem, there is an equilibrium outcome in which $n^*(c)$ trials are acquired and full revelation occurs. This must be the outcome with the highest possible ex-ante expected utility for the expert, by definition of $n^*(c)$. In this outcome, the decision precision is the same as in direct information acquisition by the decision maker. The result then follows from the observation that the expert's preferred equilibrium is by construction Pareto-efficient. Hence, in any other Pareto-efficient equilibrium, the ex-ante utility of the decision maker, which coincides with the precision of the decision, must be weakly larger than in this equilibrium.

First, notice that the condition $b \leq \left(\sqrt{1+\frac{2}{3c}}+1\right)^{-1}$ implies that $\left\lfloor \frac{1}{2b}-2 \right\rfloor \geq \left\lfloor \sqrt{\frac{2+3c}{12c}}-1.5 \right\rfloor$, that is $\widehat{n}(b) \geq n^*(c)$. This in turn implies that fully revealing the outcome of $n^*(c)$ trials is incentive compatible. Next, consider deviations at the information acquisition stage. In equilibrium, the expert's expected utility, $E\left[U^S(\mathbf{y}^*,\theta,b)\,|n\right]-c(n)$ is equal to $E\left[U^R(\mathbf{y}^*,\theta)\,|n\right]-c(n)-b^2$, the expected payoff of a decision maker who directly conducts $n^*(c)$ trials, minus $b^2$. Now suppose the expert deviates, and purchases $n'$ trials and some communication equilibrium is played in the ensuing communication subgame. Given this communication partition $P_{n'}$, $E\left[-\left(y_{p_i}^{n'}-\theta-b\right)^2|P_{n'}\right]= E\left[-\left(y_{p_i}^{n'}-\theta\right)^2|P_{n'}\right]-b^2$. If the partition played after the deviation is fully separating, then the difference between equilibrium payoff and deviation payoff is equal to the payoff difference that the decision maker would receive in the single agent decision problem if he purchased $n'$ trials rather than $n^*(c)$. This payoff difference is negative, by definition of $n^*(c)$. If some information loss occurs, the deviation gain is strictly smaller than the payoff difference that the decision maker would receive in the single agent decision problem because $E\left[-\left(y_{p_i}^{n'}-\theta\right)^2|P_{n'}\right] < E\left[U^R(\mathbf{y}^*,\theta)\,|n'\right]$ and again it is negative by the definition of $n^*(c)$. Q.E.D.

**Proof of Proposition 3.** We start from the observation that for any integer $l$, $n^*(c)=l$ for $\frac{1}{6(l+2)(l+3)} < c < \frac{1}{6(l+1)(l+2)}$, hence also for any $c$ in the interval required by the Proposition. The proof will show that if the conditions in the proposition hold, then in equilibrium the expert acquires $n^*(c)+1=l+1$ trials and fully reveals their outcome.

First, by Lemma 2, full revelation of the outcome of $l+1$ is incentive compatible for $b \leq \frac{1}{2(l+3)}$, hence it is incentive compatible for $b \leq \frac{1}{4(l+3)}$.

Next, we establish that the expert has no incentive to acquire a number of trials different from $l+1$. The expert's expected payoff from performing $l+1$ trials and fully revealing the outcome is equal to $W(l+1)=-\frac{1}{6(l+3)}-b^2$.

Because $\frac{1}{6(l+2)(l+3)} < c < \frac{1}{6(l+1)(l+2)}$ and $b \leq \frac{1}{4(l+3)}$, the proof of Proposition 5 — interchanging $n^*$ with $l+1$ —- implies that deviating from $l+1$ trials to run $n > l+1$ trials is not profitable.

By concavity of $W$, $\frac{W(l+1)-W(l-j)}{j+1} > \frac{W(l+1)-W(l-1)}{2}$. Hence, requiring that $c < \frac{W(l+1)-W(l-1)}{2} = \frac{1}{6(l+1)(l+3)}$ deters all deviations from $l+1$ to $l-j$, $j=1,...,l$.

Finally, a deviation to $l$ trials is not profitable for the expert if $c < W(l+1)-\hat{W}(l)$, where $\hat{W}(\cdot)$ was defined in the proof of Proposition 5. The rest of the proof establishes that for $l > 4$, $\frac{1}{6(l+1)(l+3)} < W(l+1)-\hat{W}(l)$, hence requiring that $c < \frac{1}{6(l+1)(l+3)}$ guarantees that the deviation to $l$ trials is not profitable. Also, it establishes that for $l < 4$, the value of $\frac{1}{6(l+1)(l+3)}-\left[W(l+1)-\hat{W}(l)\right]$ is at most $\frac{1}{3}b$ if $l=0$, $\frac{24b-1}{96}$ if $l=1$, $\frac{30b-1}{450}$ if $l=2$, $\frac{30b-1}{360}$ if $l=3$, and $\frac{63b-2}{735}$ if $l=4$, hence the condition in the proposition guarantees that $c < \min\left\{\frac{1}{6(l+1)(l+3)}, \left[W(l+1)-\hat{W}(l)\right]\right\}$.

To calculate $W(l+1) - \hat{W}(l)$, we need to compute $\hat{W}(l)$. Denoting by $y_j$ the action in the set $\left\{0, \frac{1}{l+3}, ..., \frac{l+1}{l+3}\right\}$ preferred by an expert who observed $j$ successes in $l$ trials, we obtain:

$$\begin{aligned}\hat{W}(l) &= \frac{1}{l+1}\sum_{j=0}^{l}\hat{W}\left(j,l;y_j\right) = -\sum_{j=0}^{l}\frac{1}{l+1}(y_j-b)^2 + 2\sum_{j=0}^{l}\frac{j+1}{(l+1)(l+2)}(y_j-b) - \frac{1}{3} \\ &= -\frac{1}{3} - \sum_{j=0}^{l}\frac{y_j-b}{l+1}\left[y_j - b - 2\frac{j+1}{l+2}\right].\end{aligned}$$

Hence,

$$W(l+1) - \hat{W}(l) = -\frac{1}{6(l+3)} - b^2 + \frac{1}{3} + \sum_{j=0}^{l}\frac{y_j-b}{l+1}\left[y_j - b - 2\frac{j+1}{l+2}\right]$$

$$= \frac{2k+5}{6(l+3)} - b^2 + \sum_{j=0}^{l}\frac{y_j-b}{l+1}\left[y_j - b - 2\frac{j+1}{l+2}\right].$$

Next, we characterize the expert's preferred action $y_j$, for $j = 0, ..., l$. First, we establish that $y_j \in \left\{\frac{j+1}{l+3}, \frac{j+2}{l+3}\right\}$. The payoff of type $j$ is maximized by action $\frac{j+1}{l+2} + b > \frac{j+1}{l+3}$, hence the action $\frac{j+1}{l+3}$ is preferred to any smaller action. Also, $\frac{j+1}{l+2} < \frac{j+2}{l+3}$, hence the fact that in equilibrium the type whose payoff is maximized by $\frac{j+2}{l+3} + b$ is willing to truthfully reveal his type guarantees that after a deviation to $l$ trials the action $\frac{j+2}{l+3}$ is preferred to any larger action.

Second, we observe that a sender whose payoff is maximized by $\frac{j+1}{l+2} + b$ will choose to induce action $\frac{j+1}{l+3}$ rather than $\frac{j+2}{l+3}$ if and only if $2b + \frac{2j-l}{(l+2)(l+3)} > 0$ and this quantity is increasing in $j$, hence for any bias such that $b \leq \frac{1}{4(l+3)}$, we can find a threshold $J = \left\lfloor -b(n+2)(n+3) + \frac{n}{2}\right\rfloor \leq \frac{n}{2}$ such that types $j \leq J$ prefer action $\frac{j+1}{n+3}$ and types $j > J$ prefer action $\frac{j+2}{n+3}$. Notice that $J = -1$ denotes the case where all types $j$ prefer action $\frac{j+2}{n+3}$.

Then, the difference $W(l+1) - \hat{W}(l)$ can be rewritten as

$$\begin{aligned}& W(l+1) - \hat{W}(l) \\ =\; & \frac{2k+5}{6(l+3)} - b^2 + 2\sum_{j=0}^{J}\frac{\frac{j+1}{l+3}-b}{l+1}\left(\frac{\frac{j+1}{l+3}-b}{2} - \frac{j+1}{l+2}\right) + 2\sum_{j=J+1}^{l}\frac{\frac{j+2}{l+3}-b}{l+1}\left(\frac{\frac{j+2}{l+3}-b}{2} - \frac{j+1}{l+2}\right) \\ =\; & \frac{2J^2 + 2J\left(12b - l + 10bk + 2bk^2 + 1\right) + 2 + 12b + l - 2bk + l^2 - 8bk^2 - 2bk^3}{2(l+1)(l+2)(l+3)^2}\end{aligned}$$

It is easy to check that $\frac{1}{6(l+2)(l+3)}$ is smaller than the above expression for any $J$, hence the range for $c$ identified in the statement of the proposition is nonempty.

Next, we consider the following difference:

$$W(l+1) - \hat{W}(l) - \frac{1}{6(l+1)(l+3)}$$

$$\begin{aligned} &= \frac{2J^2 + 2J\left(12b - l + 10bk + 2bk^2 + 1\right) + 2 + 12b + l - 2bk + l^2 - 8bk^2 - 2bk^3}{2(l+1)(l+2)(l+3)^2} \\ &\quad - \frac{1}{6(l+1)(l+3)} \\ &= \frac{3J^2 + J\left(36b - 3k + 30bk + 6bk^2 + 3\right) + 18b - l - 3bk + l^2 - 12bk^2 - 3bk^3}{3(l+3)^2(l+1)(l+2)} \end{aligned}$$

The denominator is positive. The numerator is a quadratic expression in $J$. For $l \geq 8$, this quadratic is positive for any $l$ and any $b$ hence $\min\left\{\frac{1}{6(l+1)(l+3)}, \left[W(l+1) - \hat{W}(l)\right]\right\} = \frac{1}{6(l+1)(l+3)}$. Using the definition of $J$, we have that:

For $l = 0$, $J = -1$ and $W(l+1) - \hat{W}(l) = \frac{1-6b}{18}$, hence $\frac{1}{6(l+1)(l+3)} - \left[W(l+1) - \hat{W}(l)\right] = \frac{b}{3}$.

For $l = 1$, if $b \leq \frac{1}{24}$, $J = 0$ and $\frac{1}{6(l+1)(l+3)} - \left[W(l+1) - \hat{W}(l)\right] = 0$. If instead $\frac{1}{24} < b < \frac{1}{16}$, then $J = -1$ and $\frac{1}{6(l+1)(l+3)} - \left[W(l+1) - \hat{W}(l)\right] = \frac{24b-1}{48}$.

For $l = 2$, if $b \leq \frac{1}{30}$ $J = 0$ and $\frac{1}{6(l+1)(l+3)} - \left[W(l+1) - \hat{W}(l)\right] < 0$. For $b \in (\frac{1}{30}, \frac{1}{20}]$, $J = 0$ and $\frac{1}{6(l+1)(l+3)} - \left[W(l+1) - \hat{W}(l)\right] = \frac{30b-1}{450}$.

For $l = 3$, if $b \leq \frac{1}{60}$ $J = 1$ and $\frac{1}{6(l+1)(l+3)} - \left[W(l+1) - \hat{W}(l)\right] < 0$. For $b \in [\frac{1}{60}, \frac{1}{24}]$, $J = 0$ and $\frac{1}{6(l+1)(l+3)} - \left[W(l+1) - \hat{W}(l)\right] = \frac{30b-1}{320}$.

For $l = 4$, if $b \leq \frac{1}{42}$ $J = 1$ and $\frac{1}{6(l+1)(l+3)} - \left[W(l+1) - \hat{W}(l)\right] < 0$. For $b \in [\frac{1}{42}, \frac{1}{28}]$, $J = 0$ and $\frac{1}{6(l+1)(l+3)} - \left[W(l+1) - \hat{W}(l)\right] = \frac{63b-2}{735}$.

For $l = 5$, if $b \leq \frac{1}{112}$, $J = 2$. If $b \in \left[\frac{1}{112} < b \leq \frac{3}{112}\right]$, $J = 1$. If $b \in [\frac{3}{112} < b \leq \frac{5}{112}]$, $J = 0$. In each of these three cases, $\frac{1}{6(l+1)(l+3)} - \left[W(l+1) - \hat{W}(l)\right] < 0$.

For $l = 6$, from the expression for $J$ one can see that either $J = 1$ or $J = 2$. In both cases, $\frac{1}{6(l+1)(l+3)} - \left[W(l+1) - \hat{W}(l)\right] < 0$.

For $l = 7$, from the expression for $J$ one can see that either $J = 1$ or $J = 2$ or $J = 3$. In all these cases, $\frac{1}{6(l+1)(l+3)} - \left[W(l+1) - \hat{W}(l)\right] < 0$.

We can therefore conclude that for $l > 4$, $\frac{1}{6(l+1)(l+3)} < W(l+1) - \hat{W}(l)$, hence requiring that $c < \frac{1}{6(l+1)(l+3)}$ guarantees that the deviation to $l$ trials is not profitable. Moreover, we have established that for $l < 4$, the value of $\frac{1}{6(l+1)(l+3)} - \left[W(l+1) - \hat{W}(l)\right]$ is at most $\frac{1}{3}b$ if $l = 0$, $\frac{24b-1}{96}$ if $l = 1$, $\frac{30b-1}{450}$ if $l = 2$, $\frac{30b-1}{360}$ if $l = 3$, and $\frac{63b-2}{735}$ if $l = 4$, hence the condition in the proposition guarantees that $c < \min\left\{\frac{1}{6(l+1)(l+3)}, \left[W(l+1) - \hat{W}(l)\right]\right\}$, hence guarantees that the deviation to $l$ trials is not profitable. Q.E.D.

**Proof of Proposition 4.** Consider $n^*(c)$, the optimal number of trials under direct information acquisition by definition in (11). To prove the Proposition it is sufficient to show that there exists an equilibrium in which the sender performs $n^*(c)$ trials and fully reveals his information in the communication stage. Such an equilibrium, if it exists, would be the expert-preferred equilibrium. So, in any Pareto-efficient equilibrium the decision-maker's expected payoff has to be (at least weakly) greater than in this equilibrium.

To establish the existence of the desired equilibrium, in which the expert runs $n^*(c)$ trials and fully reveals their realizations, first, note that the condition $b \leq \left(2\sqrt{1+\frac{2}{3c}}+2\right)^{-1}$ and definition (11) together imply that $b \leq \frac{1}{2(n^*(c)+2)}$. So, by Lemma 2 full revelation is incentive compatible at the communication stage after the expert runs $n^*(c)$ trials.

Further, the expert's expected payoff after running $n^*(c)$ trials and fully revealing their realizations is equal to $-\frac{1}{6(n^*+2)} - b^2 - cn^*$. By definition, $n^*(c) \in \arg\max_n -\frac{1}{6(n+2)} - cn$. Hence, $n^*(c) \in \arg\max_n W(n) - cn \equiv -\frac{1}{6(n+2)} - b^2 - cn$.

So, to complete the proof it is sufficient to establish that for any $n \in \{0, 1, ..., \infty\}$, $W(n) \geq \hat{W}(n)$ where $\hat{W}(n)$ is the expected payoff that the expert gets after deviating to $n$ signals.

To establish this inequality, first, note that $W(n) = \sum_{j=0}^{n} \frac{W(j,n)}{n+1}$ where

$$
\begin{aligned}
W(j,n) &= -\int_0^1 \left(E\left[\theta | j, n\right] - \theta - b\right)^2 f(\theta | j, n)\, d\theta \\
&= -\int_0^1 \left(E\left[\theta | j, n\right] - \theta\right)^2 f(\theta | j, n)\, d\theta - b^2 \\
&= -\left[(E\left[\theta | j, n\right])^2 - 2(E\left[\theta | j, n\right])\frac{j+1}{n+2} + \frac{(j+2)(j+1)}{(n+3)(n+2)}\right] - b^2 \\
&= -\left[\left(\frac{j+1}{n+2}\right)^2 - 2\left(\frac{j+1}{n+2}\right)\frac{j+1}{n+2} + \frac{(j+2)(j+1)}{(n+3)(n+2)}\right] - b^2
\end{aligned}
$$

$$
= -\left[\frac{(j+2)(j+1)}{(n+3)(n+2)} - \left(\frac{j+1}{n+2}\right)^2\right] - b^2 \tag{A8}
$$

Similarly, $\hat{W}(n) = \sum_{j=0}^{n} \frac{\hat{W}(j,n;)}{n+1}$, where

$$\hat{W}(j,n) = - \max_{y_j \in \left\{ \frac{1}{n^*+2}, \frac{2}{n^*+2}, \dots, \frac{n^*+1}{n^*+2} \right\}} \int_0^1 \left(y_j - \theta - b\right)^2 f(\theta | j, n)\, d\theta$$

$$= - \int_0^1 \left[ (y_j - b)^2 + \theta^2 - 2\theta (y_j - b) \right] \frac{(n+1)!}{j!\,(n-j)!} \theta^j (1-\theta)^{n-j}\, d\theta$$

$$= - \left[ (y_j - b)^2 + \int_0^1 \frac{(n+1)!}{j!\,(n-j)!} \theta^{j+2} (1-\theta)^{n-j}\, d\theta - 2(y_j - b) \int_0^1 \frac{(n+1)!}{j!\,(n-j)!} \theta^{j+1} (1-\theta)^{n-j}\, d\theta \right]$$

$$= - \left[ (y_j - b)^2 + \frac{(n+1)!}{j!\,(n-j)!} \frac{(2+j)!\,(n-j)!}{(n+3)!} - 2(y_j - b) \frac{(n+1)!}{j!\,(n-j)!} \frac{(1+j)!\,(n-j)!}{(n+2)!} \right]$$

(A9)

$$= - \left[ \left(y_j - b\right)^2 - 2\left(y_j - b\right) \frac{j+1}{n+2} + \frac{(j+2)(j+1)}{(n+3)(n+2)} \right].$$

Note that the message $y_j$ optimally chosen by type $j$ (i.e. the expert who observed $j$ successes in $n$ trials) has to be compatible with the equilibrium beliefs that he has acquired $n^*$ signals, even off the equilibrium path. Therefore, $y_j \in \left\{ \frac{1}{n^*+2}, \frac{2}{n^*+2}, \dots, \frac{n^*+1}{n^*+2} \right\}$.

The proof proceeds by showing that for any $j \leq n - j$

(A10)

$$D(j,n) \equiv \left[ W(j,n) - \hat{W}\left(j, n; y_j\right) \right] + \left[ W(n-j, n) - \hat{W}\left(n-j, n; y_{n-j}\right) \right] \geq 0.$$

Since types $j$ and $n-j$ are ex-ante equally likely after $n$ experiments, inequality (A10) implies that $W(n) \geq \hat{W}(n)$. [22]

Before computing $D(j,n)$ let us establish the following useful property.

**Claim A.** Suppose that $y_j = \frac{k+1}{n^*+2}$ for some $k \in \{0, 1, \dots, n^*\}$. Then either $y_{n-j} = \frac{n^*-k+1}{n^*+2}$ or $y_{n-j} = \frac{n^*-k+2}{n^*+2}$.

**Proof of Claim A:** For any $j \in \{0, 1, \dots, n\}$, define

$$k_j \in \arg \min_{k'=0,\dots,n^*} \left| \frac{k'+1}{n^*+2} - \left( \frac{j+1}{n+2} + b \right) \right|. \tag{A11}$$

If for some $j$, the maximizer $k'$ of the above expression is not unique, then choose one of the (two) maximizers arbitrarily and set it equal to $k_j$. So, $y_j = \frac{k_j+1}{n^*+2}$.

We need to distinguish two cases:

Case 1: $y_j = \frac{k_j+1}{n^*+2} \leq \frac{j+1}{n+2}$, and Case 2: $y_j = \frac{k_j+1}{n^*+2} > \frac{j+1}{n+2}$.

[22] If $n$ is odd, there is an even number of possible types $\{0, 1, \dots, n+1\}$, and $\frac{n+1}{2}$ pairs of types $(j, n-j)$ with $j \leq n - j$. If $n$ is even, then there is an odd number of possible types, and so there are $\frac{n}{2}$ pairs $(j, n-j)$ with $j < n - j$, plus the type $\frac{n}{2}$. When $j = \frac{n}{2}$, we have $n - j = j$. In this case $D\left(\frac{n}{2}, n\right) = 2\left[W\left(\frac{n}{2}, n\right) - \hat{W}\left(j, n; y_j\right)\right]$. The result then follows by showing that $D\left(\frac{n}{2}, n\right) > 0$ and that $D(j,n) > 0$ for each pair $(j, n-j)$ with $j < \frac{n}{2}$.

Let us start with Case 1. We will show that in this case, $y_{n-j} = \frac{n^*-k_j+1}{n^*+2}$.

Since $b \geq 0$, we have: $0 \leq \frac{j+1}{n+2} - \frac{k_j+1}{n^*+2} \leq \frac{k_j+2}{n^*+2} - \frac{j+1}{n+2}$. By (A11), $\left|\frac{k^j+1}{n^*+2} - \left(\frac{j+1}{n+2} + b\right)\right| \leq \left|\frac{k^j+2}{n^*+2} - \left(\frac{j+1}{n+2} + b\right)\right|$. So we have:

$$\left|\frac{n^* - k^j + 1}{n^* + 2} - \left(\frac{n - j + 1}{n + 2} + b\right)\right| = \left|\frac{j + 1}{n + 2} - \frac{k^j + 1}{n^* + 2} - b\right| \leq b + \left|\frac{j + 1}{n + 2} - \frac{k^j + 1}{n^* + 2}\right| \leq \tag{A12}$$
$$b + \left|\frac{k^j + 2}{n^* + 2} - \frac{j + 1}{n + 2}\right| = \left|b + \frac{k^j + 2}{n^* + 2} - \frac{j + 1}{n + 2}\right| = \left|\frac{n^* - k^j}{n^* + 2} - \left(\frac{n - j + 1}{n + 2} + b\right)\right|.$$

Inequality (A12) implies that type $n - j$ prefers the action $\frac{n^*-k_j+1}{n^*+2}$ associated with message $n^* - k_j$ to the action $\frac{n^*-k_j}{n^*+2}$ associated with message $n^* - k_j - 1$. This, in combination with $\frac{n^*-k_j+1}{n^*+2} \geq \frac{n-j+1}{n+2}$ and the fact that the utility function of type $n - j$ is single-peaked around the maximum $\frac{n-j+1}{n+2} + b$, $b \geq 0$, implies that type $n - j$ prefers message $n^* - k_j$ to any message lower than $n^* - k_j - 1$.

Let us now show that type $n - j$ also prefers to send message $n^* - k_j$ associated with action $\frac{n^*-k_j+1}{n^*+2}$ rather than any higher message associated with a higher action. This is immediate if $\frac{n-j+1}{n+2} + b \leq \frac{n^*-k_j+1}{n^*+2}$ If, on the other hand, $\frac{n-j+1}{n+2} + b > \frac{n^*-k_j+1}{n^*+2}$, this follows from the following facts: (i) $\frac{n-j+1}{n+2} \leq \frac{n^*-k_j+1}{n^*+2}$, so $\frac{n-j+1}{n+2} + b - \frac{n^*-k_j+1}{n^*+2} \leq b \leq \frac{1}{2(n^*+2)}$; (ii) $\frac{n^*-k_j+2}{n^*+2} - \frac{n-j+1}{n+2} - b \geq \frac{1}{n^*+2} - b \geq \frac{1}{2(n^*+2)}$, (iii) type $n - j$'s payoff function is symmetric and single-peaked at $\frac{n-j+1}{n+2} + b$.

Next, consider Case 2: $y_j = \frac{k_j+1}{n^*+2} > \frac{j+1}{n+2}$. Let us show that in this case $y_{n-j} \in \left\{\frac{n^*-k_j+1}{n^*+2}, \frac{n^*-k_j+2}{n^*+2}\right\}$.

Since $\frac{n^*-k_j+1}{n^*+2} < \frac{n-j+1}{n+2}$ and $b \geq 0$, the expert of type $n - j$ gets a strictly higher payoff from action $\frac{n^*-k_j+1}{n^*+2}$ than from any lower action. Thus, it remains to show that type $n - j$'s expected utility from action $\frac{n^*-k_j+2}{n^*+2}$ is higher than her expected utility from any higher action.

Further, note that we must have $\frac{j+1}{n+2} \geq \frac{k_j}{n^*+2}$. Otherwise, since $b \leq \frac{1}{2(n^*+2)}$, type $j$ would get a higher utility from action $\frac{k_j}{n^*+2}$ than from action $\frac{k_j+1}{n^*+2}$, which would contradict $y_j = \frac{k_j+1}{n^*+2}$.

Thus, $\frac{n-j+1}{n+2} \leq \frac{n^*-k_j+2}{n^*+2}$, and since the expected utility function of the type $n - j$ is symmetric around its maximum at $y = \frac{n-j+1}{n+2} + b$ and $b \leq \frac{1}{2(n^*+2)}$, we conclude that the type $n - j$ gets a higher expected utility from action $\frac{n^*-k_j+2}{n^*+2}$ than from any other actions. This completes the proof of Claim A.

Let us now turn back to the proof of the Proposition and compute $D(j, n)$. From (A8), (A9), (A10) we have

$$D(j,n) = \frac{(j+1)^2}{(n+2)^2} + \frac{(n-j+1)^2}{(n+2)^2} - 2b^2$$
$$+ \left[ \left(y_j - b\right)^2 + \left(y_{n-j} - b\right)^2 - 2\left(y_j - b\right)\frac{j+1}{n+2} - 2\left(y_{n-j} - b\right)\frac{n-j+1}{n+2} \right]$$

$$= \frac{(j+1)^2}{(n+2)^2} + \frac{(n-j+1)^2}{(n+2)^2} + \left[ y_j^2 + y_{n-j}^2 - 2y_j\frac{j+1}{n+2} - 2y_{n-j}\frac{n-j+1}{n+2} - 2b(y_j + y_{n-j} - 1) \right]$$

$$\text{(A13)} \qquad = \left(y_j - \frac{(j+1)}{(n+2)}\right)^2 + \left(y_{n-j} - \frac{(n-j+1)}{(n+2)}\right)^2 - 2b(y_j + y_{n-j} - 1).$$

If $y_{n-j} = \frac{n^*-k_j+1}{n^*+2}$, then $y_j + y_{n-j} = 1$, and hence by (A13) $D(j,n) = \left(y_j - \frac{(j+1)}{(n+2)}\right)^2 + \left(y_{n-j} - \frac{(n-j+1)}{(n+2)}\right)^2$. The latter expression is nonnegative.

If instead $y_{n-j} = \frac{n^*-k_j+2}{n^*+2}$, then $y_j + y_{n-j} = 1 + \frac{1}{n^*+2}$. So, by (A13),

$$D(j,n) = \left(y_j - \frac{j+1}{n+2}\right)^2 + \left(y_{n-j} - \frac{n-j+1}{n+2}\right)^2 - \frac{2b}{n^*+2}$$
$$\text{(A14)} \qquad = \left(\frac{j+1}{n+2} - \frac{k_j}{n^*+2}\right)^2 + \left(\frac{k_j+1}{n^*+2} - \frac{j+1}{n+2}\right)^2 - \frac{2b}{n^*+2}$$

In the proof of Case 2 of Claim A, we have established that $\frac{k_j}{n^*+2} \leq \frac{j+1}{n+2} \leq \frac{k_j+1}{n^*+2}$. Observe that $\frac{k_j+1}{n^*+2} - \frac{k_j}{n^*+2} = \frac{1}{n^*+2}$. So the value of the first two terms of $D(j,n)$, $\left(\frac{j+1}{n+2} - \frac{k_j}{n^*+2}\right)^2 + \left(\frac{k_j+1}{n^*+2} - \frac{j+1}{n+2}\right)^2$, depends only on $\frac{k_j+1}{n^*+2} - \frac{j+1}{n+2}$ and reaches its minimum when $\frac{j+1}{n+2} = \frac{k_j+1/2}{n^*+2}$. In this case, $\left(\frac{j+1}{n+2} - \frac{k_j}{n^*+2}\right)^2 + \left(\frac{k_j+1}{n^*+2} - \frac{j+1}{n+2}\right)^2 = \frac{1}{2(n+2)^2}$, and $D(j,n) = \frac{1}{2(n+2)^2} - \frac{2b}{n^*+2}$. Hence, $D(j,n) \geq 0$ when $b \leq \frac{1}{4(n^*+2)}$. This concludes the proof that under the given conditions on the parameters, $D(j,n) \geq 0$ hence $W(n) \geq \hat{W}(n)$. Q.E.D.

**Proof of Proposition 5** The proof of proposition 1 shows that if $b \leq \left(\sqrt{1 + \frac{2}{3c}} + 3\right)^{-1}$ and $c \leq \frac{5-\sqrt{17}}{48}$, the overt game has an equilibrium in which the partner acquiring information performs $n^*(c) + 1$ trials and then reveals them to the decision making partner, so that $E\left[-(\bar{y}-\theta)^2|\mathcal{P}\right]=E\left[-(y^*-\theta)^2|n^*(c)+1\right]$. The stronger result that

$$\begin{aligned}& E\left[-(\bar{y}-\theta)^2|\mathcal{P}\right]-E\left[-\left(y^*-\theta\right)^2|n^*(c)\right] \\ = \quad & E\left[-\left(y^*-\theta\right)^2|n^*(c)+1\right]-E\left[-\left(y^*-\theta\right)^2|n^*(c)\right]\geq[n-n^*(c)]\frac{c}{2}=c/2\end{aligned}$$

is satisfied when the cost $c$ is not too large, i.e., when

$$-\frac{1}{6(n^*(c)+3)}-\frac{c}{2}(n^*(c)+1)\geq-\frac{1}{6(n^*(c)+2)}-\frac{c}{2}n^*(c).$$

This inequality is satisfied when $c\leq\frac{1}{3(n^*(c)+3)(n^*(c)+2)}$, which is shown to always hold, using the expression for $n^*(c)$ of Lemma (4).

Likewise, the proof of proposition 3 shows that if $b\leq\frac{1}{4(n+3)}$, and $\frac{1}{6(n+2)(n+3)}<c<\frac{1}{6(n+1)(n+3)}-\max\left\{0,\left(\frac{1}{3}b\right)\mathbb{I}_{n=0},\left(\frac{24b-1}{96}\right)\mathbb{I}_{n=1},\left(\frac{30b-1}{450}\right)\mathbb{I}_{n=2},\left(\frac{30b-1}{360}\right)\mathbb{I}_{n=3},\left(\frac{63b-2}{735}\right)\mathbb{I}_{n=4}\right\}$, then the covert game has an equilibrium in which the partner acquiring information performs $n^*(c)+1$ trials and then reveals them to the decision making partner. Because $\frac{1}{6(n+1)(n+3)}\leq\frac{1}{3(n+3)(n+2)}$, the constraint that $c\leq\frac{1}{3(n^*(c)+3)(n^*(c)+2)}$ does not impose any additional constraint on the sufficient conditions of 3. ■

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REFERENCES


Aghion, Philippe, and Jean Tirole. 1997. “Formal and Real Authority in Organizations.” *Journal of Political Economy*, 105:1-29.

Ambrus, Attila, and Satoru Takahashi. 2008. “Multi-expert cheap talk with restricted state spaces.” *Theoretical Economics*. 3:1-27.

Aumann, Robert J., and Sergiu. Hart. 2003. “Long Cheap Talk.” *Econometrica*. 71:1619-1660.

Austen-Smith, David. 1993. “Interested Experts and Policy Advice: Multiple Referrals under Open Rule.” *Games and Economic Behavior*. 5:3-43.

Austen-Smith, David. 1994. “Strategic Transmission of Costly Information.” *Econometrica*. 62:955-963.

Battaglini, Marco. 2002. “Multiple Referrals and Multidimensional Cheap Talk.” *Econometrica*. 70:1379-1401.

Berlin, Leonard, and David M. Williams. 2000. “When an Expert Witness Is Not an Expert.” *American Journal of Roentgenology*. 174:1215-1219.

Battaglini, Marco. 2004. “Policy Advice with Imperfectly Informed Experts.” *Advances in Theoretical Economics*. 4:1-32.

Bolton, Patrick, and Mathias Dewatripont. 1994. "The Firm as a Communication Network." *Quarterly Journal of Economics.* 109:809–39.

Che, Yeon-Koo, and Navin Kartik. 2009. "Opinions as incentives." *Journal of Political Economy.* 117:815-860.

Crawford, Vincent P., and Joel Sobel. 1982. "Strategic Information Transmission." *Econometrica*. 50:1431-1451.

Dessein, Wouter. 2002. "Authority and Communication in Organizations." *Review of Economic Studies*. 69:811-838.

Eso, Peter, and Dezsö Szalay. 2010. "Incomplete Language as an Incentive Device." Unpublished.

Garicano, Luis. 2000. "Hierarchy and the Organization of Knowledge in Production." *Journal of Political Economy.* 108:874-904.

Gilligan, Thomas W., and Keith Krehbiel. 1987. "Collective Decision making and Standing Committees: An Informational Rationale for Restrictive Amendment Procedures." *Journal of Law, Economics and Organization*. 3:287-335.

Gilligan, Thomas W., and Keith Krehbiel. 1989. "Asymmetric Information and Legislative Rules with a Heterogeneous Committee." *American Journal of Political Science*. 33:459-90

Jones v. O'Young et al., 607 N.E.2d 224 (Ill 1992)

Krishna, Vijay, and John Morgan. 2001a. "A Model of Expertise." *Quarterly Journal of Economics*. 116:747-75.

Krishna, Vijay, and John Morgan. 2001b. "The Art of Conversation: Eliciting Information from Experts through Multi-Stage Communication." *American Political Science Review*. 95:435-52.

Krishna, Vijay, and John Morgan. 2004. "Asymmetric Information and Legislative Rules: Some Amendments." *Journal of Economic Theory*. 117:147-179.

Ivanov, Maxim. 2010. "Informational Control and Organizational Design." *Journal of Economic Theory*. 145:721-751.

Milgrom, Paul R. 1981. "Good News and Bad News: Representation Theorems and Applications." *Bell Journal of Economics*. 12:380–91.

Mitusch, Kay., and Roland. Strausz. 2005. "Mediation in situations of conáict and limited commitment." *The Journal of Law, Economics, & Organization*. 21:467-500.

Morgan, John, and Phillip C. Stocken. 2008. "Information Aggregation in Polls." *American Economic Review*. 98:864-896.

Nelson, Robert J. 1988. *Partners with Power.* Berkeley and Los Angeles: University of California Press.

Ottaviani, Marco. 2000. "The Economics of Advice." Unpublished.

Pei, Harry D. 2015. "Communication with Endogenous Information Acquisition." *Journal of Economic Theory. 160:132-149.*

Radner, Roy. 1993. "The Organization of Decentralized Information Processing." *Econometrica.* 61:1109-46.

Szalay, Dezsö. 2005. “The Economics of Clear Advice and Extreme Options.” *Review of Economic Studies*. 72:1173–1198.

Wolinsky, A. 2002. “Eliciting Information from Multiple Experts.” *Games and Economic Behavior*. 41:141-60.